A general space-time model for combinatorial optimization problems (and not only)

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Abstract

We consider the problem of defining a strategy consisting of a set of facilities taking into account also the location where they have to be assigned and the time in which they have to be activated. The facilities are evaluated with respect to a set of criteria. The plan has to be devised respecting some constraints related to different aspects of the problem such as precedence restrictions due to the nature of the facilities. Among the constraints, there are some related to the available budget. We consider also the uncertainty related to the performances of the facilities with respect to considered criteria and plurality of stakeholders participating to the decision. The considered problem can be seen as the combination of some prototypical operations research problems: knapsack problem, location problem and project scheduling. Indeed, the basic brick of our model is a variable x_{ilt} which takes value 1 if facility i is activated in location l at time t, and 0 otherwise. Due to the conjoint consideration of a location and a time in the decision variables, what we propose can be seen as a general space-time model for operations research problems. We discuss how such a model permits to handle complex problems using several methodologies including multiple attribute value theory and multiobjective optimization. With respect to the latter point, without any loss of the generality, we consider the compromise programming and an interactive methodology based on the Dominance-based Rough Set Approach. We illustrate the application of our model with a simple didactic example.

1 Introduction

Operational Research (OR) has been developing around a certain number of prototypical problems such as facility location, knapsack and scheduling (for a survey see Owen and Daskin (1998), Martello et al. (2000), Hartmann and Briskorn (2010), respectively). The classical OR approach formulates these problems in terms of optimization of a well defined objective function representing the preferences of a single Decision Maker (DM) in a deterministic context.

Despite a vast number of successful applications of OR techniques, we have to admit that real world decision problems require a broader methodology than the classical OR approaches. In this perspective one can observe that in OR it is more and more common to consider a plurality of objective functions (see e.g., Deb and Deb. (2014)) taking into account preferences of a multiplicity of stakeholders (see e.g., De Gooyert et al. (2017)) in an uncertain environment (see e.g., Gabrel et al. (2014)).

We have to observe also that many real life problems present elements of more than one prototypical OR problem. Consider, for example, the design of an urban development project in which several facilities have to be activated in different feasible locations in parallel or in a temporal sequence under some budget constraints. You can see that such a problem presents:

- elements of the knapsack problem related to the facility to be selected,
- elements of the facility location related to the position where facilities have to be placed,
- elements of the scheduling problem related to the period in which the selected facilities have to be activated.

In simple words, one can say that prototypical OR problems consider only one of the following questions:

- “what?”, which is the case of knapsack problems answering the question “what items should be selected?”,
- “where?”, which is the case of facility location problems answering the question “where facilities should be located?”,
- “when?”, which is the case of scheduling problems answering the question “when activities should take place?”.

Instead, complex real world decision problems consider simultaneously all the three above questions.

In the literature we can find models and methods to answer each of the single questions or pairs of them but not the combination of all three. The “what?” question, generated a strand of research related to the knapsack problem where one has to select which items should be inserted in a knapsack in order to optimize an objective function representing the overall profit of the items entering the knapsack while keeping the total weight of the selected items within the limited knapsack capacity. To solve the problem, in its multiobjective formulation, a good approximation of the set of solutions covering all possible trade-offs between the different objectives is identified (e.g., Captivo et al. (2003), da Silva et al. (2007), and Mavrotas et al. (2015)). This strand also includes the portfolio decision problems Salo et al. (2011) in which, given a set of feasible projects evaluated on a set of criteria, the choice of the projects to insert in the portfolio is guided by the maximization of a value function of the portfolio. Several studies have been proposed as, for example, in Badri et al. (2001), Liesio et al. (2007), Liesio et al. (2008) and Morton et al. (2016). Often the aim is to define a new methodology to tackle these problems as in Argyris et al. (2011), Barbati et al. (2017) or Lourenço et al. (2017).

If we are attempting to answer the "where?" question, we are formulating a facility location problem that consists in positioning a set of facilities in a given space. Usually the facilities have to satisfy some demand from the customer and the position of the facilities is determined on the basis of some objective functions representing the satisfaction of the demand Eiselt and Laporte (1995). This strand of research is wide and many models concerning different aspects of the problems have been proposed (for some interesting reviews see Drezner and Hamacher (2001), Laporte et al. (2016) or Owen and Daskin (1998)). The classical objective is the minimisation of the sum of the distances between the users and the facilities Hakimi (1964) but several other modifications have been proposed taking into account a huge disparity of objectives and several different constraints modeling different aspects of the problem (for a list see Farahani et al. (2010)).

Alternatively, considering the "when?" question, we are dealing with a scheduling problem consisting in defining the time in which to start different activities. The classical objective is to find a feasible schedule so that the project duration has to be minimized Habibi er al. (2018). Also in this case many models have been proposed, considering different aspects of the problem such as capacity constraints Koulinas et al. (2014) or robustness of the problem Abbasi et al. (2006).

In the literature combinations of two of the three questions have also been considered. For example, combining the "what?" question and the "where?" question, Ishizaka er al. (2013) selected the position for casinos in London, while Özcan et al. (201) dealt with a warehouse location selection and Tzeng et al. (2002) with a restaurant location selection problem. Cheng and Li (2004) proposed a binary integer linear programming model to determine the locations for fixed investment as construction projects. Montibeller et al. (2009) dealt with this type of problem considering multiple criteria and multiple stakeholders. Furthermore, the combination of the questions "when?" and "where?" has originated a flourishing strand of research related to the dynamic facility location problems (for a survey see Arabani and Farahani (2012)). In these problems the facilities are located in each time period of a finite planning horizon Kelly and Marucheck (1984). Many modifications have been proposed as the possibility of relocating the facilities Melo et al. (2006) or the possibility of both relocating and/or changing the activated facilities and their capacities through the time in order to satisfy the customers demand Correia and Melo (2017). Finally, the combination of "what?" and "when?" questions is modeled in several papers in which the portfolio decision problem concerns also the timing in which each project should be developed Ghasemzadeh and Archer (2000). Again, several algorithms and methods have been proposed. For example Dickinson et al. (2001) and Zuluaga et al (2007) introduced the interdependency of the projects, while Doerner et al. (2006) considered the benefits derived by the projects divided in categories. Furthermore, Ghorbani and Rabbani (2009) supposed that the projects can start in some periods and continuing or not over following periods in order to maximise the benefits derived from the portfolio and the balance of the resources allocated in the different periods. Recently, Pérez et al. (2017) modeled synergies and incompatibilities among projects and uncertainty in the parameters of the problem.

The above literature review shows that, despite very often the nature of real life problems is in between of several prototypical OR problems, there is not a general model permitting a systematic analysis of such complex problems. In view of this, we propose a general methodology permitting to handle problems that have elements of the knapsack problem, of the facility location problem and the scheduling problem at the same time. We adopt a multiobjective optimization approach to take into account a plurality of criteria as it seems natural in this type of problems. Moreover, we consider also the possibility to take into consideration uncertainty related to different potential scenarios and the presence of a plurality of stakeholders, as this can be useful in several real life contexts.

Since the problem we are handling requires answers to the basic question "what?" of the knapsack problem, considering also the questions "where?" and "when?", the methodology we are proposing defines a space-time model in which the activation of each facility is characterized not only in terms of spatial coordinates typical of location problems, but also in terms of a time frame considered in scheduling problems. From the formal point of view, the basic idea is to consider variables of the type x_{ilt} taking value 1 if facility i is activated in location l at time t, and 0 otherwise. Observe that our space-time model is not restricted to the above considered problems of facility location planning Cheng and Li (2004), but it can be applied in other relevant situations such as, for example, a project portfolio selection Montibeller et al. (2009) in which, beyond the set of selected projects, it is considered the timing with which the projects have to be realized. Observe also, that, even if the three prototypical OR problems we considered are of combinatorial optimization nature, one can always relax the binary constraints permitting the decision variables to take a value on the non-negative reals. In this way our space-time model can be applied in problems that do not require combinatorial optimization, such as the typical problems of environmental planning Huang er al.

(2011).

The paper is organized in the following way. In Section 2 we present the formulation of our space-time model. In Section 4 we illustrate a didactic example for our model while in Section 5 we apply two different multiobjective optimization methods. In Section 6 we explain how the model can be used in presence of uncertainty and plurality of stakeholders, while Section 7 concludes the paper.

2 The proposed model

The considered problem concerns a set of facilities $I = \{1, \ldots, i, \ldots, n\}$ to be placed in a set of feasible locations $L = \{1, \ldots, l, \ldots, m\}$ in different periods $T = \{0, \ldots, t, \ldots, p\}$. Each facility is evaluated with respect to a set of criteria $J = \{1, \ldots, j, \ldots, q\}$. The evaluation of facility $i \in I$ activated in location $l \in L$ with respect to the criterion $j \in J$ is denoted by $y_{ijl} \in \mathbb{R}^+$. For the sake of simplicity, without the loss of generality, we suppose that all criteria $j \in J$ are of the gain type, that is, the greater y_{ijl}, the better the evaluation of facility $i \in I$ on criterion $j \in J$ in location $l \in L$. For each period $t \in T$ a discount factor $v(t)$, with $0 \leq v(t) \leq 1$ and v being a non increasing function of t, is defined in order to discount the evaluation of performances $y_{ijl}, i \in I, j \in J, l \in L$ in future periods. The values $v(t)$, $t \in T$, have to represent the intertemporal preferences of the DM. There is a vast literature on discounting and time preference (see Frederick et al. (2002) for a survey) and, of course, among the many models proposed, that one that can be considered the more convenient with respect to the application at hand can be applied in our framework. In general, for the sake of simplicity, in the rest of the paper, when we refer to a specific discount factor $v(t)$ we consider the model presented in Samuelson (1937) and characterized by a constant interest rate ρ, such that $v(t) = (1+\rho)^{-t}$. Once defined the discounted factors $v(t), t \in T$, $V_{ijlt} = y_{ijl} \cdot v(t)$ gives the value in period 0 of the performance in period t of facility $i \in I$ activated in location $l \in L$ with respect to criterion $j \in J$. Here we are supposing that the benefit of facilities with respect to considered criteria does not depend on the time passed from their activation. Of course, this assumption is rather strong and can be relaxed considering a benefit depending also on the time passed from the activation, so that we have to consider an evaluation y_{ijlr} of facility i activated in location l with respect to the criterion j after $r, r = 1, \ldots, p-1$, periods from its activation. In this case, if the facility is activated in period τ, the discounted value in 0 of the evaluation y_{ijlr} is given by $V_{ijl\tau r} = y_{ijlr} \cdot v(\tau+r)$. Since we need to aggregate performances on different criteria, we have to consider a weight $w_j \geq 0$ such that $w_1 + \ldots + w_q = 1$, for each criterion $j \in J$, to make homogeneous their performances and performing their sum. Each facility $i \in I$ has also a cost $c_i \in \mathbb{R}^+$. The available budget for each period $t \in T$ is denoted by B_t.

The following decision variables can be considered to define the adopted strategy:

$$x_{ilt} = \begin{cases} 1, & \text{if facility } i \in I \text{ is activated in location } l \text{ in period } t \in T - \{p\}; \\ 0, & \text{otherwise.} \end{cases}$$

For example, having a set of facilities $I = \{1, 2\}$, a set of locations $L = \{1, 2\}$ and a set of periods $T = \{0, 1, 2\}$ we have to consider the following vector decision variables:

$$\mathbf{x} = [x_{110}, x_{111}, x_{120}, x_{121}, x_{210}, x_{211}, x_{220}, x_{221}].$$

If we have

$$x_{110} = x_{111} = x_{120} = 0, x_{121} = x_{210} = 1, x_{211} = x_{220} = x_{221} = 0,$$

then the adopted strategy consists in placing facility 1 in location 2 in period 1 and facility 2 in location 1 in period 0. Observe that not all 0-1 vectors $\mathbf{x} = [x_{ilt}]$ are feasible. Indeed, some constraints have to be satisfied such:

- budget constraints for which in each period $t \in T$ the expenses cannot be greater than the available budget B_t
$$\sum_{i \in I} c_i \sum_{l \in L} x_{ilt} \leq B_t, \quad \forall t \in T, \tag{1}$$
- activation constraints for which each facility can be activated at most once
$$\sum_{l \in L, t \in T} x_{ilt} \leq 1, \quad \forall i \in I. \tag{2}$$

Of course, other constraints can be considered such as precedence constraints for which some facilities cannot be activated before other related facilities have been activated. Moreover, also the budget constraints and the activation constraints can be weakened or strengthened. For example, with respect to the budget constraints, one can imagine that it is possible to lend some capital or to use the monetary return of some facility already activated. Also activation constraints can have different formulations such as no more than a fixed number of facilities of a given type can be activated.

Given a strategy $\mathbf{x}$, the benefit of criterion $j \in J$ in period $t \in T - \{0\}$ from facility $i \in I$ is obtained if i has been activated not later than period $t - 1$, otherwise it is null. Therefore the performance of facility $i \in I$ with respect to criterion $j \in J$ in location $l \in L$ at time $t \in T - \{0\}$ is

$$y_{ijlt}^{IJLT}(\mathbf{x}) = \sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl}.$$

Discounting the performance $y_{ijlt}^{IJLT}(\mathbf{x})$ we get

$$\widehat{y}_{ijlt}^{IJLT}(\mathbf{x}) = y_{ijlt}^{IJLT}(\mathbf{x})v(t) = \sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl} v(t).$$

Given a strategy $\mathbf{x}$, from the values $y_{ijlt}^{IJLT}(\mathbf{x})$ other interesting values can be obtained. First, we list the performances related to three elements among one facility $i \in I$, one criterion $j \in J$, one location $l \in L$ and one period $t \in T - \{0\}$ as follows:

- the overall performance of criterion $j \in J$ in location $l \in L$ in period $t \in T - \{0\}$, that is

$$y_{jlt}^{JLT}(\mathbf{x}) = \sum_{i \in I} y_{ijlt}^{IJLT}(\mathbf{x}) = \sum_{i \in I} \sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl},$$

- the overall performance of facility $i \in I$ in location $l \in L$ in period $t \in T - \{0\}$ taking into account all criteria, that is

$$y_{ilt}^{ILT}(\mathbf{x}) = \sum_{j \in J} \sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl},$$

- the overall performance of facility $i \in I$ with respect to criterion $j \in J$ in period $t \in T - \{0\}$ taking into account all locations, that is

$$y_{ijt}^{IJT}(\mathbf{x}) = \sum_{l \in L} \sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl},$$

- the overall performance of facility $i \in I$ with respect to criterion $j \in J$ in location $l \in L$ taking into account all periods $t \in T - \{0\}$, that is

$$y_{ijl}^{IJL}(\mathbf{x}) = \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl}.$$

In Table 1, for each of the above listed performances, we show for which set among I, J, L and T, one considers one element only and for which sets one instead considers all the elements, so that, for example, with respect to performance $y_{ijl}^{IJL}(\mathbf{x})$, the facility $i \in I$, the criterion $j \in J$ and the location $l \in L$ are fixed and, instead, all the periods $t \in T$ are comprehensively considered. We name this first set as performances of group A.

Second, we list the performances related to two elements among one facility $i \in I$, one criterion $j \in J$, one location $l \in L$ and one period $t \in T - \{0\}$, as follows:

Table 1: Performances of group A.

Performance	Facilities	Criteria	Locations	Periods
$y_{jlt}^{JLT}(\mathbf{x})$	All facilities	Criterion j	Location l	Period t
$y_{ilt}^{ILT}(\mathbf{x})$	Facility i	All criteria	Location l	Period t
$y_{ijt}^{IJT}(\mathbf{x})$	Facility i	Criterion j	All locations	Period t
$y_{ijl}^{IJL}(\mathbf{x})$	Facility i	Criterion j	Location l	All periods

- the overall performance of the strategy $\mathbf{x}$ in location $l \in L$ at time $t \in T - \{0\}$, that is

$$y_{lt}^{LT}(\mathbf{x}) = \sum_{i \in I} \sum_{j \in J} \sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl}$$

- the overall performance of strategy $\mathbf{x}$ with respect to criterion $j \in J$ at time $t \in T - \{0\}$ considering all locations, that is

$$y_{jt}^{JT}(\mathbf{x}) = \sum_{i \in I} \sum_{l \in L} \sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl},$$

- the overall performance of strategy $\mathbf{x}$ with respect to criterion $j \in J$ in location $l \in L$ taking into account all periods $t \in T - \{0\}$, that is

$$y_{jl}^{JL}(\mathbf{x}) = \sum_{i \in I} \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl},$$

- the overall performance of facility $i \in I$ in period $t \in T - \{0\}$ considering all criteria $j \in J$ and all locations $l \in L$, that is

$$y_{it}^{IT}(\mathbf{x}) = \sum_{j \in J} \sum_{l \in L} \sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl},$$

- the overall performance of facility $i \in I$ in location $l \in L$ considering all criteria $j \in J$ and all periods $t \in T - \{0\}$, that is

$$y_{il}^{IL}(\mathbf{x}) = \sum_{j \in J} \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl},$$

- the overall performance of facility $i \in I$ with respect to criterion $j \in J$ considering all locations

$l \in L$ and all periods from $t \in T - \{0\}$, that is

$$y_{ij}^{IJ}(\mathbf{x}) = \sum_{l \in L} \sum_{t \in T-\{0\}} \sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl}.$$

Table 2: Performances of group B.

Performance	Facilities	Criteria	Locations	Periods
$y_{lt}^{LT}(\mathbf{x})$	All facilities	All criteria j	Location l	Period t
$y_{jt}^{JT}(\mathbf{x})$	All facilities	Criterion j	All locations	Period t
$y_{jl}^{JL}(\mathbf{x})$	All facilities	Criterion	Location l	All periods
$y_{it}^{IT}(\mathbf{x})$	Facility i	All criteria	All locations	Period t
$y_{il}^{IL}(\mathbf{x})$	Facility i	All criteria	Location l	All periods
$y_{ij}^{IJ}(\mathbf{x})$	Facility i	Criterion j	All locations	All periods

In Table 2, with respect to the above introduced performances, it is showed for which two sets among I, J, L and T, one considers one element only and for which two sets, instead, one considers all the elements, so that, for example, with respect to performance $y_{ij}^{IJ}(\mathbf{x})$, the facility $i \in I$ and the criterion $j \in J$ are fixed and, instead, all the locations $l \in L$ and all the periods $t \in T$ are comprehensively considered. We name this second set as performances of group B.

Third, we list all the performances related to one elements among one facility $i \in I$, one criterion $j \in J$, one location $l \in L$ and one period $t \in T - \{0\}$, as follows

- the overall performance in period $t \in T - \{0\}$ considering all facilities $i \in I$, all criteria $j \in J$ and all locations $l \in L$, that is

$$y_t^T(\mathbf{x}) = \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl},$$

- the overall performances of strategy $\mathbf{x}$ in location $l \in L$ considering all criteria $j \in J$ and all periods $t \in T - \{0\}$, that is

$$y_l^L(\mathbf{x}) = \sum_{i \in I} \sum_{j \in J} \sum_{t \in T-\{0\}} \sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl},$$

- the overall performance with respect to criterion $j \in J$ considering all facilities $i \in I$, all locations $l \in L$ and all periods $t \in T - \{0\}$, that is

$$y_j^J(\mathbf{x}) = \sum_{i \in I} \sum_{l \in L} \sum_{t \in T-\{0\}} \sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl},$$

- the overall performance of facility $i \in I$ considering all criteria $j \in J$, all locations $l \in L$ and all periods $t \in T - \{0\}$, that is

$$y_i^I(\mathbf{x}) = \sum_{j \in J} \sum_{l \in L} \sum_{t \in T-\{0\}} \sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl}.$$

In Table 3, with respect to the above introduced performances, it is showed for which set among I, J, L and T one considers one element only and for which three sets, instead, one consider all the elements, so that, for example, with respect to performance $y_i^I(\mathbf{x})$, the facility $i \in I$ is fixed and, instead, all the criteria $j \in J$, all the locations $l \in L$ and all the periods $t \in T$ are comprehensively considered. We name this third set as performances of group C.

Table 3: Performances of group C.

Performance	Facilities	Criteria	Locations	Periods
$y_t^T(\mathbf{x})$	All Facilities	All criteria	All locations	Period t
$y_l^L(\mathbf{x})$	All facilities	All criteria	Location l	All periods
$y_j^J(\mathbf{x})$	All facilities	Criterion j	All locations	All periods
$y_i^I(\mathbf{x})$	Facility i	All criteria	All locations	All Periods

Finally, we can list the overall performance of strategy $\mathbf{x}$ taking into account all facilities $i \in I$, all criteria $j \in J$, all locations $l \in L$ and all periods $t \in T - \{0\}$ as

$$y(\mathbf{x}) = \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T-\{0\}} \sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl}.$$

Let us point out that all the above performances can be discounted. For example the discounted value at time 0 of $y_{jlt}^{JLT}(\mathbf{x})$ is given by

$$\widehat{y}_{jlt}^{JLT}(\mathbf{x}) = y_{jlt}^{JLT}(\mathbf{x}) v(t) = \sum_{i \in I} \sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl} v(t).$$

We shall denote by $\widehat{y}_{indices}^{sets}(\mathbf{x})$ the discounted value of the corresponding non-discounted performance

$y^{sets}_{indices}(\mathbf{x})$, so that $\widehat{y}^{JLT}_{jlt}(\mathbf{x})$ is the discounted value of $y^{JLT}_{jlt}(\mathbf{x})$, $\widehat{y}^{ILT}_{ilt}(\mathbf{x})$ is the discounted value of $y^{ILT}_{ilt}(\mathbf{x})$, and so on.

In the first instance, the problem is to define the strategy $\mathbf{x}$ giving the maximum overall discounted performance $\widehat{y}(\mathbf{x})$ subject to the constraints of the problem such as the budget constraints and the activation constraints.

However, the above model permits to take into account a great plurality of performances $y^{sets}_{indices}(\mathbf{x})$ and $\widehat{y}^{sets}_{indices}(\mathbf{x})$ constituting a rich dashboard that can be very meaningful for the DM. In fact, the DM can fix some constraints in terms of minimal requirements of performances $y^{sets}_{indices}(\mathbf{x})$ and $\widehat{y}^{sets}_{indices}(\mathbf{x})$. More in general, we can handle the whole model in terms of optimization of performances $y^{sets}_{indices}(\mathbf{x})$ and $\widehat{y}^{sets}_{indices}(\mathbf{x})$. We shall explore this possibility in Section 5.

2.1 A possible extension with continuous variables

Our model can work also when the variables x_{ilt} are defined in $\mathbf{R}^+$. In this case the variables can be defined as the amount of budget that has been allocated to facility of type i in location l at period t. In this case binary constraints (2) must not be considered, while to the original budget constraint (1), we can add additional budget constraints. In particular, we can define:

- $B^{\leq}_{it}$ as the maximum budget to be allocated to facility $i \in I$ in period $t \in T$,
- $B^{\geq}_{it}$ as the minimum budget to be allocated to facility $i \in I$ in period $t \in T$,
- $B^{\leq}_{lt}$ as the maximum budget to be allocated to location $l \in L$ in period $t \in T$,
- $B^{\geq}_{lt}$ as the minimum budget to be allocated to location $l \in L$ in period $t \in T$,

so that for each of the above quantities we can define the additional constraints:

- in period $t \in T$, no more than the maximum budget $B^{\leq}_{it}$ can be allocated to facility $i \in I$:

$$\sum_{l \in L} x_{ilt} \leq B^{\leq}_{it}, \tag{3}$$

- in period $t \in T$, no less than the minimum budget $B^{\geq}_{it}$ must be allocated to facility $i \in I$:

$$\sum_{l \in L} x_{ilt} \geq B^{\geq}_{it}, \tag{4}$$

- in period $t \in T$, no more than the maximum budget $B^{\leq}_{lt}$ can be allocated to location $l \in L$:

$$\sum_{i \in I} x_{ilt} \leq B^{\leq}_{lt}, \tag{5}$$

- in period $t \in T$, no less than the minimum budget $B^{\geq}_{lt}$ must be allocated to location $l \in L$:

$$\sum_{i \in I} x_{ilt} \geq B^{\geq}_{lt}. \tag{6}$$

Those budgets, and the associated constraints, are not necessarily defined for all the facilities $i \in I$ and for all the locations $l \in L$. To ensure the feasibility of the model, it should also be verified that for each $i \in I$ then $B^{\leq}_{it} \leq B_t$ and for each $l \in L$ then $B^{\leq}_{lt} \leq B_t$. Let us underline that also for the continuous case we can handle the whole model in terms of multiobjective optimization of the performances $y^{sets}_{indices}(\mathbf{x})$ and $\widehat{y}^{sets}_{indices}(\mathbf{x})$.

3 Potential Applications

In this section we aim to show the the strengths and the versatility of our model through the description of some of its potential applications.

The typical application of our model is in urban planning problems, where it can be adopted to plan the location and the time to build and to activate urban infrastructures devoted to provide the necessary services to the citizens, e.g. leisure centres or healthcare services Farahani et al. (2018). The decisions related to the location of urban services can be effectively supported by our model at the point that it could be recognized as a useful tool of the so called computational urbanism Verebes (2013), that is, the set of computational techniques adopted to assist urban planning. In this perspective, different strength points of our approach can be identified as follows:

- the project portfolio formulation: the facilities to be located are often of different types and the majority of the real case studies are related to the location of a portfolio of facilities instead of one single facility Arabani and Farahani (2012).

- the multiobjective approach: the simultaneous decision of the location and the time in which establish a facility, that is a characteristic of the dynamic facility location problem Chardaire et al. (1996), can be expressed in a more realistic form by considering a plurality of objective functions Nickel and da Gama (2015) including new type of objectives, rather than considering the usual mono-objective approach Farahani et al. (2010);

- the consideration of the uncertainty and risk aspects: the capacity to allow for the unforeseen circumstances required in the urban planning design Verebes (2013) can be taken into account through the definition of different probabilistic scenarios;

- the handling of the dynamic aspects: the effects and consequences of urban projects are in general not limited to a single time period Correia and Melo (2017) but they are distributed on different periods, so that they can be very naturally represented in terms of multiperiod contributions;

- the possibility to take into account constraints of different nature: for example, the need of rationalizing the public services, always increasing due to the reduction of the available budgets in the last years Cavola et al. (2018), can be modeled with constraints requiring the closure of some facilities;

- the intrinsic interactivity of the procedure: beyond the potential adoption of a formal interactive methodology (see Section 5.2) permitting the DM to construct the proposed strategy in a participated way Edelenbos and Klijn (2005) even revising its own preferences during the process, the consideration of so many performance indices permits the DM to take under control the consequences of the decision with respect to different points of view, so that one can become aware of possible modifications and improvements of the plan.

To discuss the application of our model in a real life problem, let us consider a typical problem of urban design as the definition of a waste management system (for a review see e.g. Achillas et al. (2006)). Recently, Eiselt and Marianov (2014) proposed a bi-objective model to choose the size and the positions of landfills and transfer stations among a set of potential locations. Their problem could be approached with our model considering facilities of different types (landfills, transfer stations or recycling center or other facilities included in the waste management systems) and of different size. In this perspective, using our model, dedicated and specialized constraints, related for example to the consideration of special environmental zones and geology of the soil, could be easily added to the budget constraints. Moreover, our space-time model permits to handle some issues not considered in the original model. First, we can stress the importance of treating the whole system as a portfolio in which the choices are made considering the interaction and the contribution of all type of facilities Eiselt and Marianov (2014). Second, being our model multiobjective, we could include a variety of objectives, as for example reduction of pollution or strengthening of the sustainability that are vastly becoming the most important objectives to be considered Bing et al. (2017). Third, we could deal with the uncertainty that such a problem presents in relation, for example, to the quantity and the composition of the garbage produced Yadav et al. (2018). Fourth, our approach can model the temporal distribution of the activation of facilities assuring that the required reliability of the long term planning will be satisfied Wang et al. (2018). Fifth, our model can take into consideration constraints and objectives related to the need for rationalizing the waste management services and make them more affordable for the councils permitting to plan the closure of some of the facilities in order to make the management of the system more efficient Silva et al. (2017). Finally, the introduction of interactive multiobjective methodologies can help to make a participated decision taking adequately into account the perspectives of the different stakeholders in order to guarantee openness and transparency to the public Eiselt and Marianov (2014).

Similar considerations can be carried out for the definition of an healthcare systems (for a review see, e.g., Daskin and Dean (2005)). In this case several type of facilities need to be located ranging from hospitals to the local surgeries. A variety of objectives can be considered in this application in which often the location of the facilities is handled as a network Mousazadeh et al. (2018), or adopting a hierarchical model Smith et al. (2013). Our model could be used in planning these services given that it can deal with facilities of different types and levels. Several objectives could be considered, including equal accessibility to the facilities for all the patients Hu et al. (2018). In addition, probabilistic aspects can be taken into account especially in applications characterized by a relevant uncertainty Dehe and Bamford (2015). Moreover, the dynamic nature of our model permits to take into account the required adaptability through the time of the plan Ahmadi-Javid et al. (2017), permitting to tackle scenarios where the number of healthcare facilities tends to be rationalized Bruno et al. (2018) and a specific resilience to unforeseen changes is needed Hanefeld et al. (2018). Finally, our model could help to deal with challenges derived by such a complex problem Afshari (H.), providing a tool that allows the different stakeholders to single out the contribution of the different facilities in the different areas and in the different periods.

Some other possible applications for our model could be:

- the definition of charging points for bikes, car, taxi and buses Liu et al. (2018) to locate in several areas of the cities that could present a different demand Csonka and Csiszár (2017),
- the location of facilities in a supply chain system, especially to guarantee the required integration of the locations of the different types of facilities as portfolio of projects, to take into consideration the multiplicity of objective functions and the scheduling constraints characterizing such a problem Melo et al. (2009) and to integrate routing and stochastic considerations in the decision model Govindan et al. (2017).

4 Illustrative example

We illustrate the proposed model with the following hypothetical decision problem. Let us suppose that a council is expected to decide which public interest facilities should be activated in the next 5 years, choosing between two possible locations available for each of them. In particular, we consider an example involving the following eight desirable facilities $I = \{1, \ldots, 8\}$:

Table 4: Evaluations on the three criteria in each location and associated costs for the eight facilities considered in the illustrative example.

Facilities	*EconomicImpact*		*SocialImpact*		*EnvironmentalImpact*		Cost
	North	South	North	South	North	South	
School	21	23	90	80	23	32	200
Leisure Centre	36	46	59	72	36	34	300
Council Offices	18	20	22	30	21	26	150
Recycling Centre	60	65	71	60	90	88	100
Start Up Incubator	80	82	12	12	15	12	150
Healthcare Centre	20	18	19	19	45	59	200
Community Centre	35	31	56	48	33	40	100
Social Housing	12	21	69	73	18	17	250

- School, $i = 1$,
- Leisure Centre, $i = 2$,
- Council Offices, $i = 3$,
- Recycling Centre, $i = 4$,
- Start Up Incubator, $i = 5$,
- Healthcare Centre, $i = 6$,
- Community Centre, $i = 7$,
- Social Housing, $i = 8$,

evaluated in terms of the following three criteria $J = \{1, \ldots, 3\}$:

- Economic impact, $j = 1$,
- Social impact, $j = 2$,
- Environmental impact, $j = 3$.

We suppose to have two different locations $L = \{1, 2\}$ in which the facilities can be sited, named North ($l = 1$) and South ($l = 2$). For the sake of the simplicity, we give an evaluation of each facility on each criterion and for each location on a scale [0,100] (see Table 4). We assume that the evaluation does not depend on the period. Note, however, that our model can deal also with evaluations that change through the time and with any type of quantitative evaluations. Moreover,

Table 5: Budget available in each period

Year	Budget
Start	400
First Year	100
Second Year	200
Third Year	200
Fourth Year	150

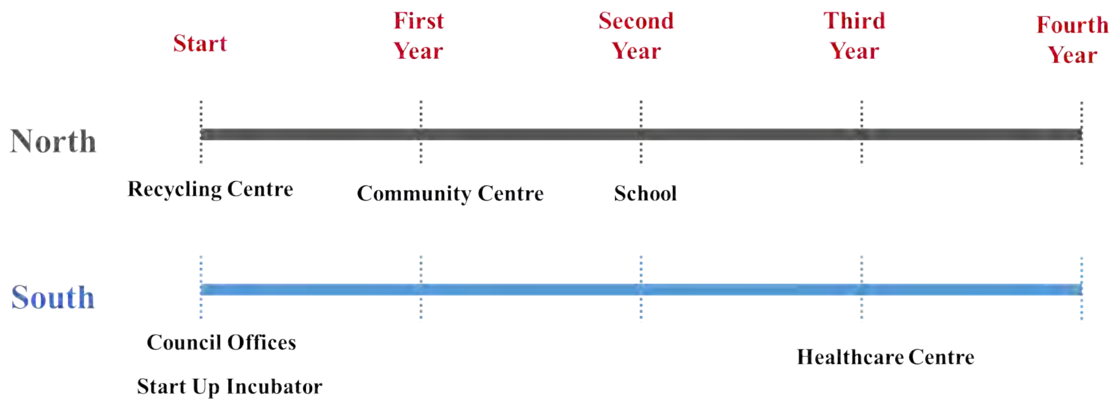

Figure 1: Optimal solution obtained by maximization of the overall performance.

each facility has an associated opening cost (in thousand Euro) which is also reported in Table 4. The available budget (in thousand Euro) is given for each period as detailed in Table 5. In addition, the interest rate is supposed to be equal to 0.1 for all the periods. The council is setting up the plans for the next 5 years $T = \{0, 1, \ldots, 5\}$ deciding which investments pursuit. We define a weight for each criterion and, in particular, $w_1 = 0.5$ for the economic impact, $w_2 = 0.3$ for the social impact and $w_3 = 0.2$ for the environmental impact.

Using the commercial software CPLEX v.12.1, we find the vector $\mathbf{x}$ that maximises the objective function $\widehat{y}(\mathbf{x})$ subject to the budget constraint. We also suppose that each facility can be activated only once, e.g., each facility cannot be activated in two different locations and in two different periods.

We obtain the following decision variables equal to 1: $x_{112}, x_{320}, x_{410}, x_{520}, x_{623}, x_{711}$, meaning that:

- The facility *School* is scheduled to be activated in location *North* at the beginning of the *second* year;
- The facility *Council Offices* is scheduled to be activated in location *South* at the beginning of the *start* year;
- The facility *Recycling Centre* is scheduled to be activated in location *North* at the beginning of the *start* year;

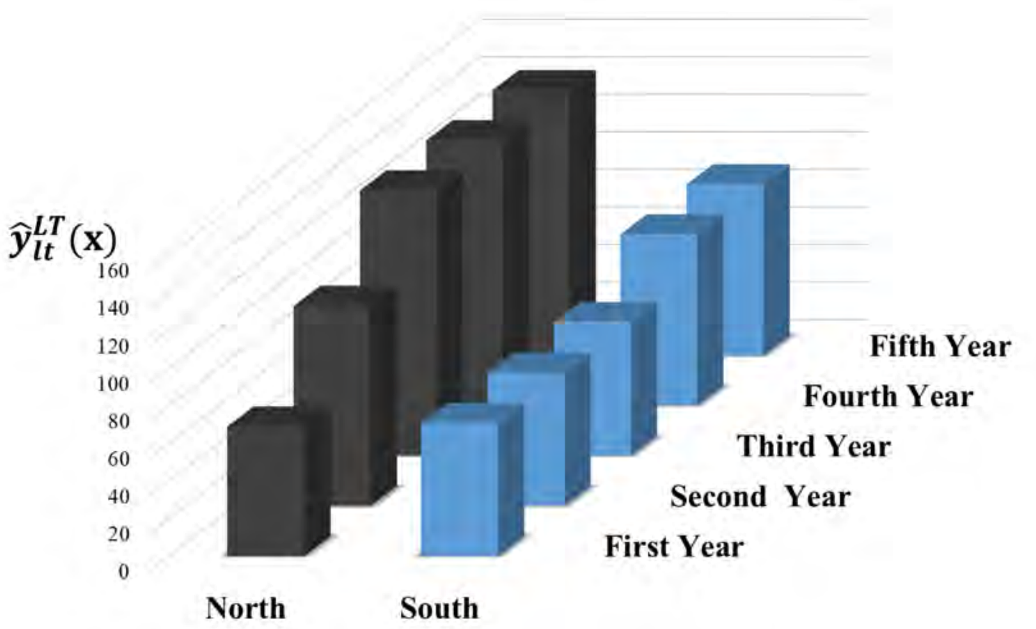

Figure 2: Distribution along the time of the performances for the optimal strategy in the two different locations.

- The facility *Start Up Incubator* is scheduled to be activated in location *South* at the beginning of the *start* year;
- The facility *Healthcare Centre* is scheduled to be activated in location *South* at the beginning of the *third* year;
- The facility *Community Centre* is scheduled to be activated in location *North* at the beginning of the *first* year.

The other facilities (Leisure Centre and Social Housing) have not been activated given the available budget constraint. The optimal strategy is reported in Figure 1.

The performances $\widehat{y}^{sets}_{indices}(\mathbf{x})$ that we have described before can be summarized in a series of graphs. These graphs can help understanding the solution and, especially, can help the DM to visualize the performance corresponding to the optimal solution da Silva et al. (2017). Indeed, these charts can be used to compare potential Pareto solutions in a multiobjective context, supporting the intuition of the DM, and making the model more appealing even for high level managers often inhibited from adopting more sophisticated and complex decision support models Ghasemzadeh and Archer (2000). For the sake of space we present the most representative charts.

First, let us show in Figure 2 the performance of the strategy suggested to the council (i.e., the optimal solution to our time - space model) in the two locations North and South. In this case we

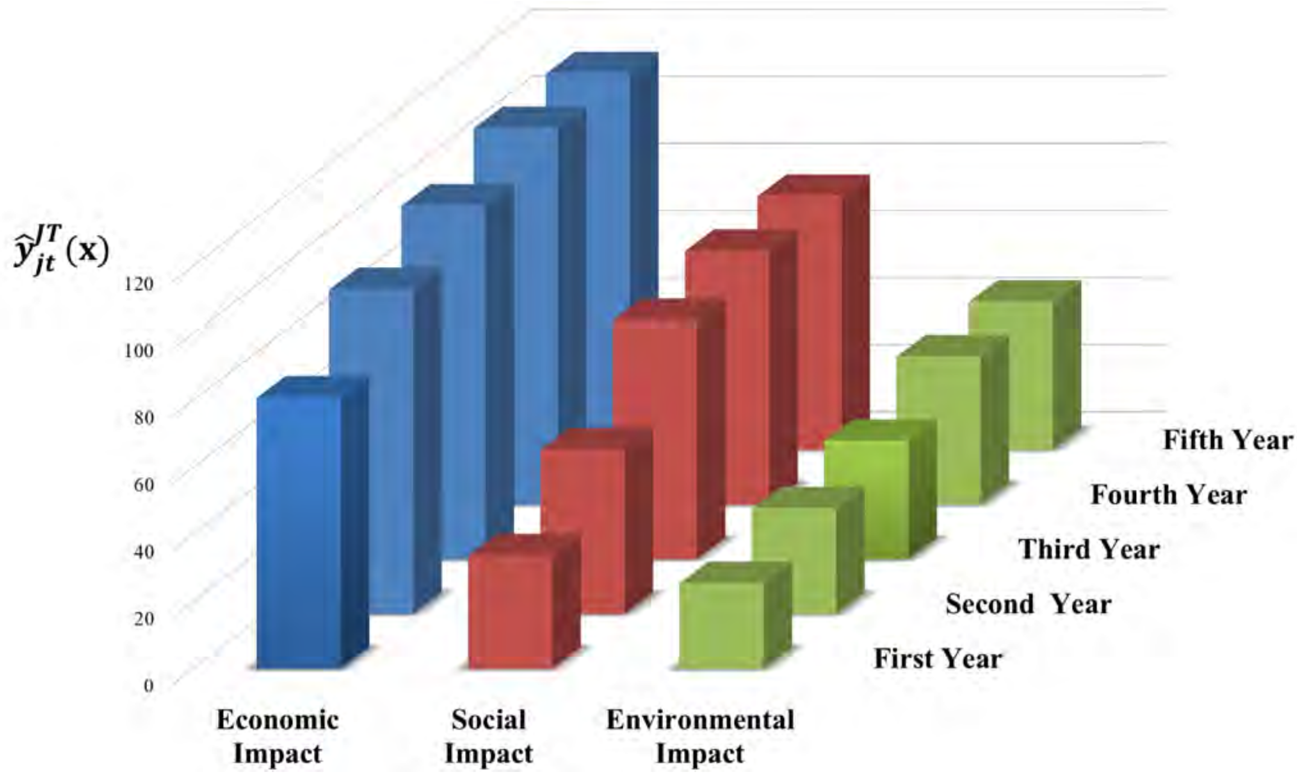

Figure 3: Time distribution of the performances for the optimal strategy with respect to each criterion.

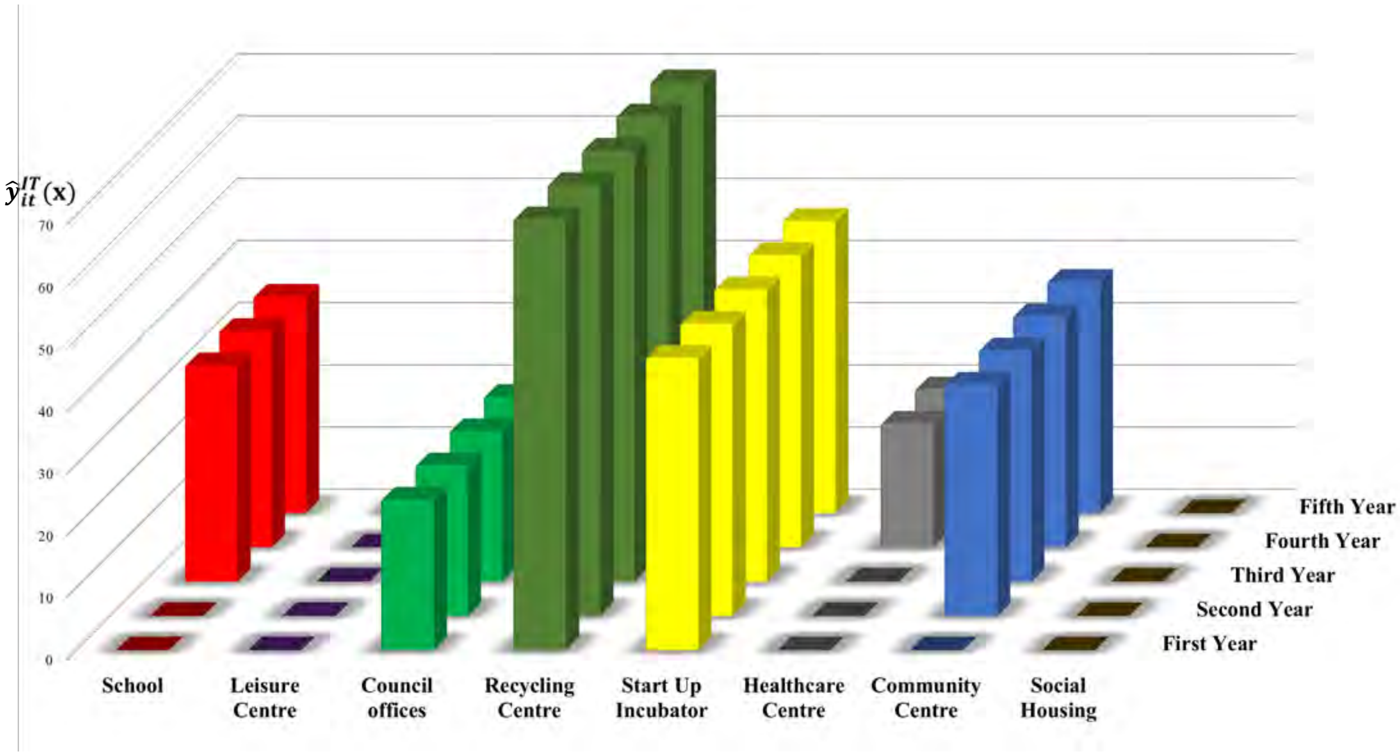

Figure 4: Time distribution of the performances for the facilities in the optimal strategy.

are recording the $\widehat{y}_{lt}^{LT}(\mathbf{x})$ (on the $y - axis$) in each period and in each location. It is possible to note that the performance assumes a bigger value in the North than in the South. Also, there is an increasing of the performance in the time at a greater pace in location North than in location South. Note that the performances are defined excluding period 0, that represents the start of our planning horizon. While at $t = 0$ we can define decision variables, the performance of the adopted plan will be evaluated only at the beginning of the first year. Moreover, the performances are discounted so that we can compare the contribution of performances obtained in different periods.

In Figure 3 we report the performance $\widehat{y}_{jt}^{JT}(\mathbf{x})$ of the optimal strategy with respect to each criterion and through the time. We can see that Economic impact has a greater importance for the solution given the highest bars and its bigger increase through the time. Note that the performances in this chart have not been weighted. This allows a neat comparison without the influence of particular weights adopted. In Figure 4 we summarize the performance of each activated facility in the optimal solution through the time. We are representing $\widehat{y}_{it}^{IT}(\mathbf{x})$, i.e., the performance of each activated facility through the time. Indeed, the *Recycling Centre* is contributing more than the other facilities. The DM could be interested in detailing the contribution of these facilities for each criterion, in the location North where it has been activated (see Figure 5). The biggest contribution is provided by the criterion Environmental Impact. In this case we are reporting the $\widehat{y}_{jt}^{JT}(x_{410})$.

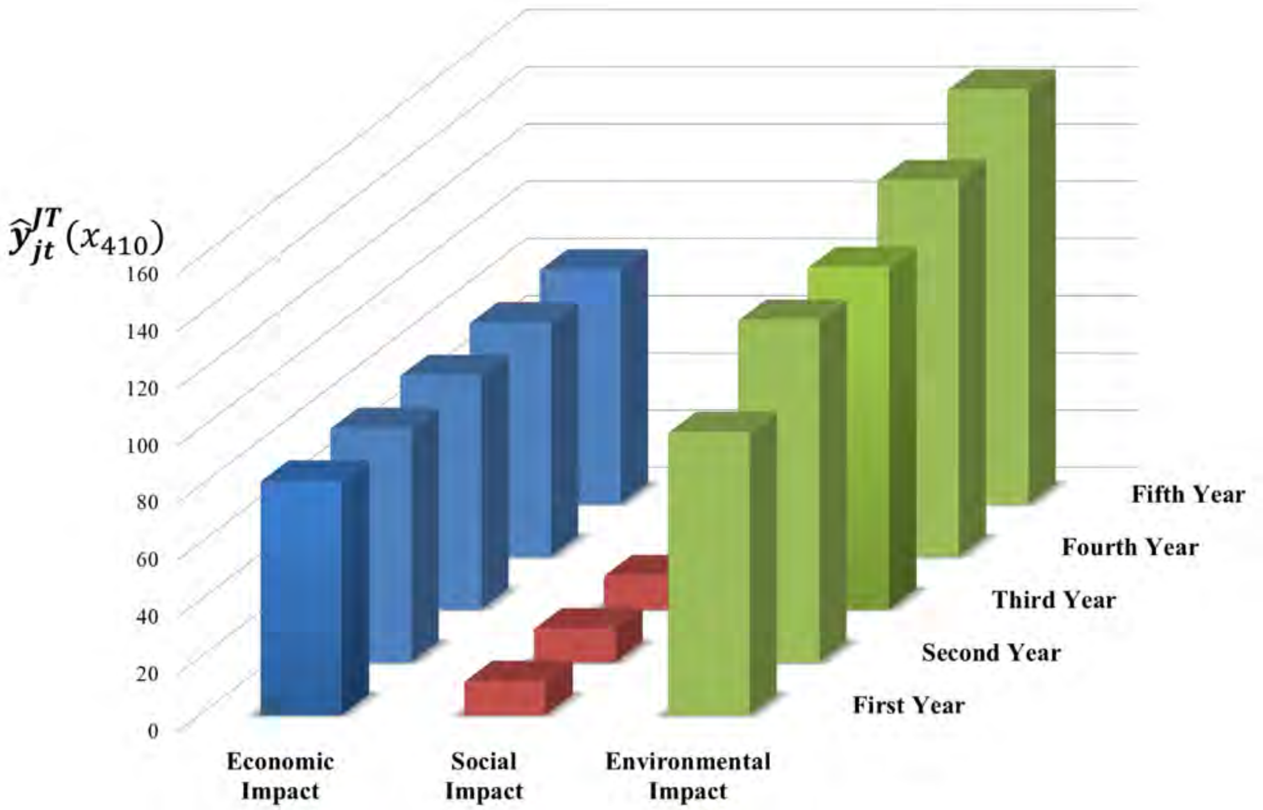

Figure 5: Time distribution of the performances for the facility *Recycling Centre* with respect to each criterion.

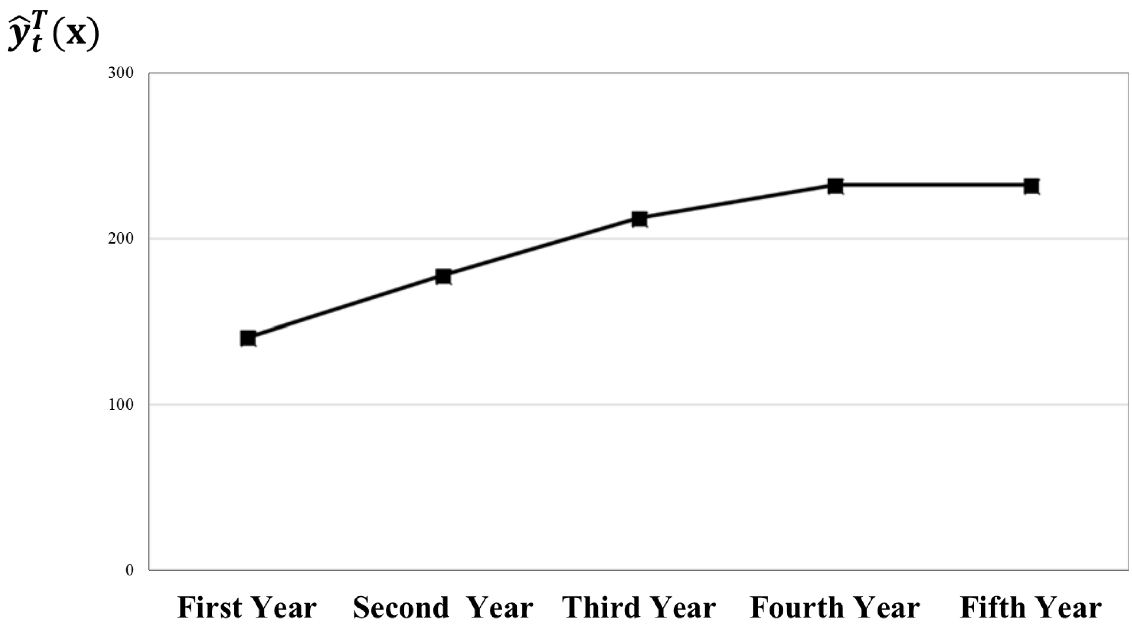

Figure 6: Time distribution of the overall performances.

Table 6: Budget available for each facility in each period

Facilities	Start		First Year		Second Year		Third Year		Fourth Year	
	$B^{\leq}_{i0}$	$B^{\geq}_{i0}$	$B^{\leq}_{i1}$	$B^{\geq}_{i1}$	$B^{\leq}_{i2}$	$B^{\geq}_{i2}$	$B^{\leq}_{i3}$	$B^{\geq}_{i3}$	$B^{\leq}_{i4}$	$B^{\geq}_{i4}$
School	-	10	50	-	-	5	20	-	-	-
Leisure Centre	70	20	150	5	10	10	-	-	-	16
Council Offices	-	-	-	3	8	8	-	-	-	-
Recycling Centre	32	16	-	-	-	-	2	2	260	5
Start Up Incubator	-	-	70	5	140	10	-	2	14	-
Healthcare Centre	-	8	-	-	-	4	-	-	-	2
Community Centre	30	-	-	5	-	-	10	1	16	14
Social Housing	-	16	60	-	180	10	-	-	-	-

Table 7: Budget available for each location in each period

Locations	Start		First Year		Second Year		Third Year		Fourth Year	
	$B^{\leq}_{l0}$	$B^{\geq}_{l0}$	$B^{\leq}_{l1}$	$B^{\geq}_{l1}$	$B^{\leq}_{l2}$	$B^{\geq}_{l2}$	$B^{\leq}_{l3}$	$B^{\geq}_{l3}$	$B^{\leq}_{l4}$	$B^{\geq}_{l4}$
North	65	6	-	2	-	-	4	4	13	10
South	21	3	10	7	6	6	20	5	-	-

Finally, in Figure 6 we can summarize the overall performance of the optimal strategy provided by the activated facilities through the time, indicating on the $y - axis$ the $\widehat{y}_t^T(\mathbf{x})$. This graph can help DMs to visualize the increase through the time of the contribution of all facilities, for all criteria and for each location. For this solution we can highlight that the increasing has a similar pace for the first four years while is less strong in the final year.

4.1 Illustrative Example: Continuous case

For the continuous case, the values of $B^{\geq}_{it}$, $B^{\leq}_{it}$, $B^{\geq}_{lt}$ and $B^{\leq}_{lt}$ are reported in Tables 6 and 7, respectively. Those budgets are not defined in all the cases (when no value is defined for the budget, and so no associated constraint is defined, the symbol "$-$" is reported in the Tables). These values are used in the formulation of constraints (5) and (6). Adopting, as before, the weighted approach, we obtain the variables different from 0.

We obtain the temporal distribution of the budget between facilities and locations shown in Table 8. For example, the variable $x_{110} = 10$, means that 10 is the budget allocated for the activation of *School* in location North at the start of the planning period; the variable $x_{112} = 5$ means that 5 is the budget allocated to the activation of *School* in the first year in location South, and so on.

Graphs and charts analogous to those ones reported for the combinatorial model can be provided also in this case.

Table 8: Budget available for each location in each period

Decision Variables (DV)	Values	DV	Values	DV	Values	DV	Values
x_{110}	10	x_{321}	3	x_{511}	3	x_{624}	2
x_{112}	5	x_{322}	8	x_{513}	1	x_{711}	5
x_{220}	20	x_{410}	32	x_{521}	2	x_{713}	1
x_{221}	5	x_{411}	82	x_{522}	10	x_{714}	14
x_{222}	10	x_{412}	153	x_{523}	1	x_{810}	15
x_{223}	195	x_{413}	2	x_{610}	8	x_{820}	1
x_{224}	16	x_{414}	118	x_{622}	4	x_{822}	10

5 Multiobjective methodologies for the space - time model

In this Section we propose to use two different multiobjective methodologies to find the most preferred solution for the DM to the space - time model. We first propose an introduction for the two methodologies and their associated strengths and, after, we expose the results of the two methodologies when applied to our illustrative example introduced in Section 4. In this way, the reader can see the advantages and the differences derived by the applications of these two methodologies. Several algorithms, mainly exact, have been provided in the literature to find solutions to multi-objective 0-1 linear programming problems (for a review, see Ehrgott et al. (2016)). When dealing with small problem instances, some algorithms can look for an approximation of the whole set of efficient solutions. These include the branch and bound algorithms Przybylski and Gandibleux (2017) or the ϵ constraint method Cohon (2013); Mavrotas and Florios (2013). Some interactive algorithms integrate optimization procedures (i.e., Alves and Climaco et al. (2014); Argyris et al. (2011); Mavrotas and Diakoulaki (1998)) with the aim of singling out the set (possibly a singleton) of the most preferred solutions for the DM. In the same perspective, other methods suggest the adoption of a linear value approach (see, e.g., Salo et al. (2011)) or the use of a goal programming procedure Jones and Tamiz (2016). In what follows, we illustrate how our space - time model can be handled with two multiobjective methodologies. First we consider a classical approach called Compromise Programming (CP) Romero (2001) adopted to solve several multiobjective optimization models. The second approach is more recent and takes into account the preferences of the DM using an interactive procedure. It has been proposed by Greco et al. (2008) and applied to portfolio decision problems in Barbati et al. (2017).

5.1 Compromise Programming

In a CP approach the aim is to minimize the maximum deviation from the ideal point, i.e., the point with the best evaluation. For our model we characterize three types of CP approaches considering three different ideal points.

First, in the Compromise Programming for Location **(CPL)** we characterize our target as the vector $\widehat{y}^{L^*} = [\widehat{y}_l^{L^*}]$ where, for each $l \in L$, $\widehat{y}_l^{L*}$ represents the best actualized performance that can be attained by location l, that is

$$\widehat{y}_l^{L^*} = \max_{\mathbf{x}} \widehat{y}_l^L(\mathbf{x}) = \sum_{i\in I}\sum_{j\in J}\sum_{t\in T-\{0\}}\sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl} v(t).$$

Different metrics can be adopted in order to define the closeness of the obtained strategy to the ideal point. Following Drezner et al. (2006), in order to get a balanced solution, we minimize the maximum relative deviation $\Delta_l^L(\mathbf{x})$ on the set of locations $l \in L$, defined as

$$\Delta_l^L(\mathbf{x}) = \frac{\widehat{y}_l^{L^*} - \widehat{y}_l^L(\mathbf{x})}{\widehat{y}_l^{L^*}}.$$

Then, the distance of the strategy $\mathbf{x}$ from the ideal point is $\Delta^L(\mathbf{x}) = \max_{l\in L} \Delta_l^L(\mathbf{x})$. This optimisation strategy could suit several DMs. In our example the council could be interested in attempting to minimize the differences among the locations so that the optimal solution is $\mathbf{x}^* = \arg\min \Delta^L(\mathbf{x})$.

Second, we specify what we call the Compromise Programming for Objectives **(CPO)** where the target is the vector $\widehat{y}^{J^*} = [\hat{y}_j^{J^*}]$ where for each $j \in J$, $\widehat{y}_j^{J^*}$ represents the best actualized performance that can be attained on criterion j, that is,

$$\widehat{y}_j^{J^*} = \max_{\mathbf{x}} \widehat{y}_j^J(\mathbf{x}) = \sum_{i\in I}\sum_{l\in L}\sum_{t\in T-\{0\}}\sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl} v(t).$$

Analogously to the previous case, we shall minimize the maximum relative deviation $\Delta_j^J(\mathbf{x})$, on the set of criteria $j \in J$, defined as

$$\Delta_j^J(\mathbf{x}) = \frac{\widehat{y}_j^{J^*} - \widehat{y}_j^J(\mathbf{x})}{\widehat{y}_j^{J^*}}.$$

Then, the distance of the strategy $\mathbf{x}$ from the ideal point is $\Delta^J(\mathbf{x}) = \max_{j\in J} \Delta_j^J(\mathbf{x})$. DMs adopting such an optimization strategy would like to balance the importance of all the criteria so that the optimal solution is $\mathbf{x}^* = \arg\min \Delta^J(\mathbf{x})$.

Lastly, we can define what we call Compromise Programming for Objectives and Location **(CPOL)** where our target is

$$\widehat{y}_{jl}^{JL^*} = \max_{\mathbf{x}} \widehat{y}_{jl}^{JL}(\mathbf{x}) = \sum_{i\in I}\sum_{t\in T-\{0\}}\sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl} v(t).$$

Table 9: Position and activation period of the facilities in the best solutions obtained with the different CP approaches.

Facilities	**CPL**		**CPO**		**CPOL**	
	Location	Period	Location	Period	Location	Period
School	North	2	North	2	North	3
Leisure Centre	-	-	-	-	South	0
Council Offices	South	0	South	0	-	-
Recycling Centre	North	0	North	0	North	0
Start Up Incubator	North	0	South	0	North	4
Healthcare Centre	South	3	South	3	South	2
Community Centre	North	1	South	1	South	1
Social Housing	-	-	-	-	-	-

Again, we shall minimize the maximum relative deviation $\Delta_{jl}^{JL}(\mathbf{x})$, on the set of criteria $j \in J$ and on the set of locations $l \in L$, defined as

$$\Delta_{jl}^{JL}(\mathbf{x}) = \frac{\widehat{y}_{jl}^{JL^*} - \widehat{y}_{jl}^{JL}(\mathbf{x})}{\widehat{y}_{jl}^{JL^*}}.$$

Then, the distance of the strategy $\mathbf{x}$ from the ideal point is $\Delta^{JL}(\mathbf{x}) = \max_{j\in J, l\in L} \Delta_{jl}^{JL}(\mathbf{x})$. This last case is a combination of the first two compromise optimization approaches and attempts to balance the differences from the ideal points for both the criteria and the locations $\mathbf{x}^* = \arg\min \Delta^{JL}(\mathbf{x})$.

Illustrative Example: Compromise Programming

We apply the three compromise optimization approaches described above to our illustrative example introduced in Section 4. We obtain the optimal compromised strategies reported in Table 9. For each CP approach we noted the location and the period in which a facility has been activated; the symbol "$-$" means that a facility has not been activated.

In Figure 7 we report the overall performance of the optimal strategy obtained with each of the CP approaches with respect to each criterion, while, in Figure 8, we show the overall performance with respect to each location. We can see that CPO gives quite balanced values with respect to the overall performances $\widehat{y}_j^J$ on considered criteria $j \in J$, while the performances $\widehat{y}_l^L$ with respect to locations $l \in L$ result quite unbalanced. This is because this strategy does not search for a compromise in the values of the differences between the two locations. Nevertheless, CPL and CPOL have similar results. In particular, CPL obtains a solution with very balanced values $\widehat{y}_j^J$ between North and South, while CPOL shows a good balance for both criteria and locations.

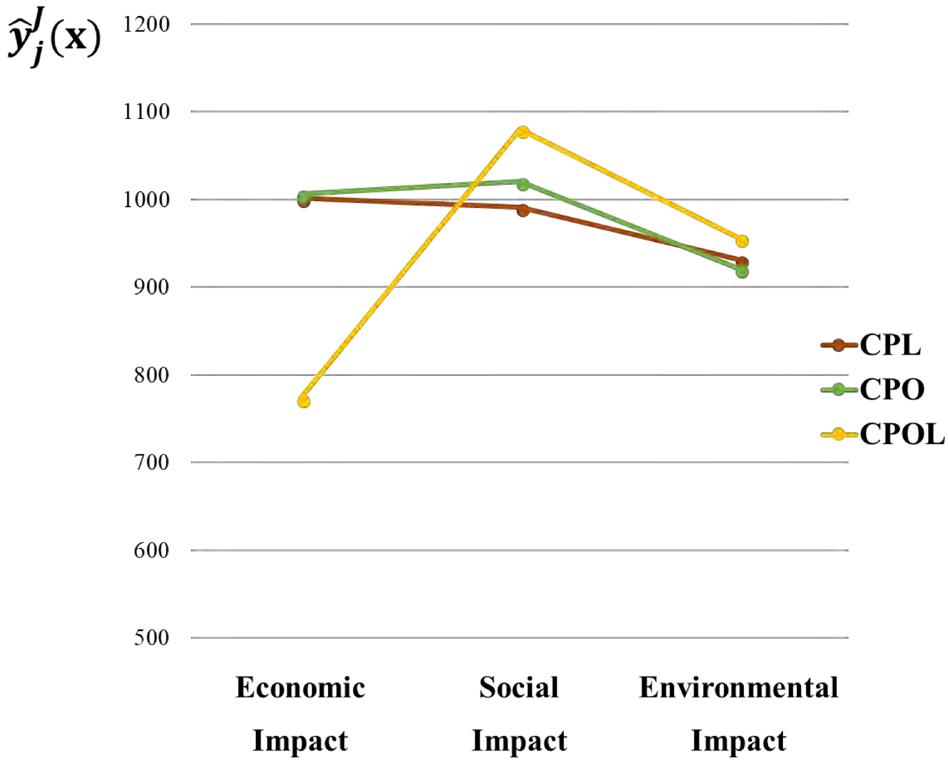

Figure 7: The performances $\widehat{y}_j^J$ of the best strategies obtained for each CP approach.

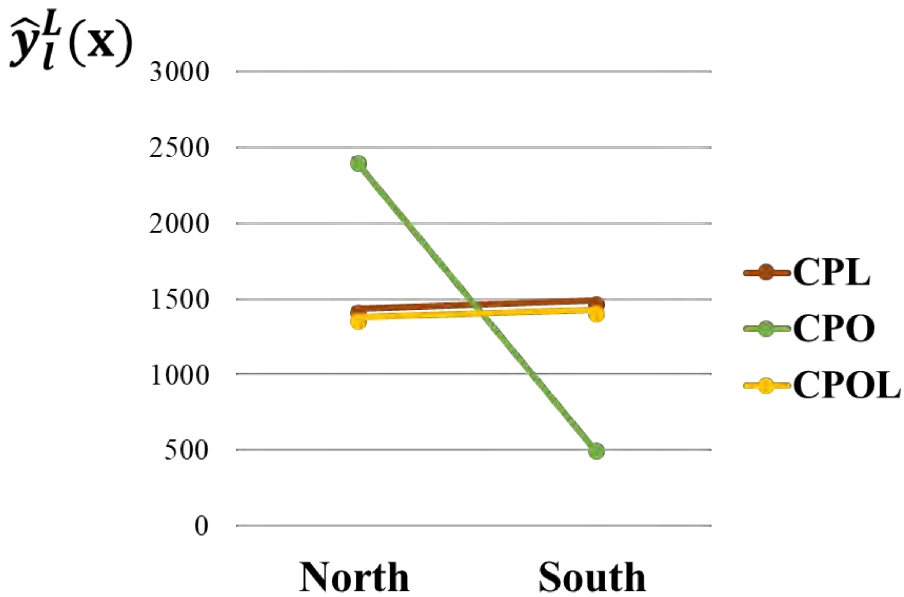

Figure 8: The performances $\widehat{y}_l^L$ of the best strategies obtained for each CP approach.

5.2 A multiobjective interactive optimization approach

Interactive Multi-objective Optimization (IMO) methods (for a survey, see Miettinen et al. (2008)) look for a solution being as much as possible satisfactory for the DM through procedures alternating computation phases (in which multiobjective optimization problems are solved), and dialogue phases (in which preference information is collected from the DM). IMO methods are among the most common used in the multiobjective theory. They are characterized by several strength points as:

- focusing only on solutions of interest for the DM reducing the computation effort Phelps and Koksalan (2003);
- specifying and adjusting the DM preferences between each iteration throughout the whole process Miettinen et al. (2008);
- integrating contrasting opinions from several stakeholders Soltani et al. (2015);
- adapting to disparate application contexts (e.g. data mining Bandaru et al. (2017), manufacturing Chica et al. (2016), chemical process design Miettinen and Hakanen (2017));
- the capability of readapting to the changing conditions or redefinition of some of them as can be required, for example, when deciding to urban services location in a masterplan Verebes (2013);
- the capability of these techniques to deal with uncertain contributions Jiang et al. (2018).

Among the many IMO methods proposed in the literature, we shall take into consideration a method called IMO-DRSA Greco et al. (2008) but, of course, any other IMO method can be applied as well. In the IMO-DRSA an interactive procedure is integrated in a multiobjective optimization procedure with the use of the Dominance Based Rough Set Approach (DRSA) (see, e.g., Greco et al. (2016, 2001, 2010)). The main idea is that the DM is provided with a list of feasible solutions and he is asked to select one if he is convinced that it is completely satisfactory. In this case the procedure ends. On the contrary, the DM is asked to indicate a set of relatively good solutions in the list, so that a binary partition into classes "good" and "others" of the list of proposed solutions is obtained. From such indirect preference information, using the DRSA, we induce a set of "if ..., then ..." decision rules explaining the partition in "good" and "others" in terms of values $g_j(\mathbf{x})$ taken for strategy $\mathbf{x}$ by the criteria $g_j, j = 1, \ldots, m$, considered in the multiobjective optimization (for example, in our space-time model performances of group A, B, C or the overall performance $y(\mathbf{x})$). More precisely, supposing that all criteria g_j have an increasing direction of preference, the rules are logical statements of the type

$$\text{if } g_{j_1}(\mathbf{x}) \geq \rho_{j_1} \text{ and } g_{j_2}(\mathbf{x}) \geq \rho_{j_2} \text{ and } \ldots \text{ and } g_{j_r}(\mathbf{x}) \geq \rho_{j_r}, \text{ then } \mathbf{x} \text{ is a good strategy.}$$

The decision rules so obtained are presented to the DM, that is asked to select the rule he considers the most representative of his preferences. The selected decision rule gives a set of constraints

$$g_{j_1}(\mathbf{x}) \geq \rho_{j_1},\ g_{j_2}(\mathbf{x}) \geq \rho_{j_2},\ \ldots,\ g_{j_r}(\mathbf{x}) \geq \rho_{j_r}$$

to be added to the current set, so that the solution space is consequently reduced in a region of feasible strategy being more appealing for the DM. From the current set of feasible strategies, another set of representative strategies is built and presented to the DM, so that the cycle starts again, until the DM finds a satisfactory strategy.

The IMO-DRSA allows to combine the advantages of an interactive method with the advantages of the Dominance Rough Set Theory Greco et al. (2001) that has shown to be very effective in the analysis of preference ordered data. Many times it has been proved that this is an effective method that can deal with real world applications (e.g., chakar et al. (2016); Chen and Tsai (2016) especially with the numerous extensions that have been proposed through the years (e.g. Luo et al. (2018)).

We can summarize the advantages derived by the application of this method as follows:

- the decision rules serve as synthetic description of the DM's opinions expressed in simple and understandable terms;

- a strategy is considered of a good quality only when a specific number of projects with a at least a given evaluation on each criterion have been scheduled to be implemented;

- the clear language of the rules permits the DM to easily indicate those rules that best represent his preferences;

- the final solution is found by a process of progressive inclusion of more and more demanding thresholds that gradually constrain the set of strategies satisfying these requirements;

- the process continues until the DM considers a strategy as fully satisfactory in terms of the quality of projects included.

To apply IMO-DRSA to the specific decision problem represented by our space time model, following Barbati et al. (2017), the performances $\hat{y}^{sets}_{indices}$ are all transformed in qualitative ordinal evaluations by means of suitable thresholds. With this aim, for each $j \in J$ and each $l \in L$, the DM is asked to define a set $S_{j,l}$ consisting of $J(h)$ thresholds

$$S_{j,l} = \{s_{1,j,l}; \ldots; s_{J(h),j,l} : s_{1,j,l} < s_{2,j,l} < \ldots < s_{J(h),j,l}\},$$

Table 10: Satisfaction levels for the three criteria in both the locations considered in the illustrative example.

Satisfaction levels	*EconomicImpact*	*SocialImpact*	*EnvironmentalImpact*
s_1: Satisfactory	20	20	20
s_2: Very satisfactory	35	35	35
s_3: Extremely satisfactory	55	55	55

permitting to define a set $\mathcal{C}_h$ consisting of $J(h)+1$ qualitative satisfaction classes $C_{a,j,l}$

$$\mathcal{C}_h = \{C_{1,j,l}, \ldots, C_{J(h)+1,j,l}\}$$

such that the greater $a = 1, \ldots, J(h)+1$, the more preferred is the project from class $C_{a,j,l}$. The facilities $i \in I$ are assigned to satisfaction classes $C_{a,j,l} \in \mathcal{C}_h$ according to the following rule: for all $i \in I$

- facility i is assigned to class $C_{1,j,l}$ if $y_{ijl} < s_{1,j,l}$;
- facility i is assigned to class $C_{a,j,l}$ with $a = 2, \ldots, J(h)$, if $s_{a-1,j,l} \leq y_{ijl} < s_{a,j,l}$;
- facility i is assigned to class $C_{J(h)+1,j,l}$ if $s_{J(h),j,l} \leq y_{ijl}$.

Illustrative Example: definition of qualitative evaluations

The council has defined three satisfaction levels for each criterion (see Table 10). We suppose the same levels are considered for all criteria in all the locations. In this way we can define our satisfaction classes: "weakly satisfactory", "satisfactory", "very satisfactory", and "extremely satisfactory".

For each facility $i \in I$ with respect to each location $l \in L$ and to each criterion $j \in J$ we have:

- facility i is "weakly satisfactory" if $y_{ijl} < 20$;
- facility i is "satisfactory" if $20 \leq y_{ijl} < 35$;
- facility i is "very satisfactory" if $35 \leq y_{ijl} < 55$;
- facility i is "extremely satisfactory" if $55 \leq y_{ijl}$.

For each strategy $\mathbf{x}$, in each location $l \in L$, for each criterion $j \in J$, each satisfaction level $s_{a,j,l} \in S_{j,l}$, we consider the set of facilities attaining threshold $s_{a,j,l}$:

$$P_{a,j,l}(\mathbf{x}) = \{i \in I : y_{ijl}(\mathbf{x}) \geq s_{a,j,l}\}.$$

In simple words, considering the qualitative scale given in the above example, with respect to criterion j, for the strategy $\mathbf{x}$,

- $P_{1,j,l}(\mathbf{x})$ is the set of satisfactory facilities,
- $P_{2,j,l}(\mathbf{x})$ is the set of very satisfactory facilities,
- $P_{3,j,l}(\mathbf{x})$ is the set of extremely satisfactory facilities.

For the sake of simplicity, in what follows we shall write $|P_{a,j,l}(\mathbf{x})|$, as $F_{a,j,l}(\mathbf{x})$.

We can consider the following three main formulations of our space-time multiobjective optimization problem:

- a *location-oriented multiobjective optimization* in which the objective functions are the sums on all considered criteria of the number of activated facilities attaining an evaluation of at least level $a, a = 1, \ldots, h$ in a given location $l \in L$, that is, in the above example, for each facility $i \in I$:
  - the number of facilities at least satisfactory for the first criterion plus the analogous number for the second criterion and so on until the last criterion;
  - the number of facilities at least very satisfactory for the first criterion plus the analogous number for the second criterion and so on until the last criterion;
  - the number of facilities extremely satisfactory for the first criterion plus the analogous number for the second criterion and so on until the last criterion.

  Therefore the location oriented multiobjective optimization problem can be formulated as

  $$\max \sum_{j \in J} F_{a,j,l}(\mathbf{x}), \ \forall l \in L, \ \forall s_{a,j,l} \in S_{j,l}$$

  under the constraints (1) and (2), and the other possible constraints of the original problem;

- a *criterion oriented multiobjective optimization* in which the objective functions are the sums on all considered locations of the number of activated facilities of at least level $a, a = 1, \ldots, h$, for a given criterion $j \in J$; that is, in the above example, for each criterion $j \in J$:
  - the number of facilities at least satisfactory in the first location plus the analogous number in the the second location and so on until the last location;

- the number of facilities at least very satisfactory in the first location plus the analogous number in the the second location and so on until the last location;
- the number of facilities extremely satisfactory for the first location plus the analogous number for the second location and so on until the last location.

Therefore the criterion oriented multiobjective optimization problem can be formulated as

$$\max \sum_{l \in L} F_{a,j,l}(\mathbf{x}),\ \forall j \in J,\ \forall s_{a,j,l} \in S_{j,l},$$

under the constraints (1) and (2), and the other possible constraints of the original problem;

- a *criterion and location oriented multiobjective optimization* in which the objective functions are combinations of one location $l \in L$, one criterion $j \in J$ and the number of activated facilities of at least level $a, a = 1, \ldots, h$; that is, in the above example, for each criterion $j \in J$ and $l \in L$:
  - the number of facilities at least satisfactory;
  - the number of facilities at least very satisfactory;
  - the number of facilities extremely satisfactory.

  Therefore the criterion and location oriented multiobjective optimization problem can be formulated as

  $$\max F_{a,j,l}(\mathbf{x}),\ \forall j \in J, \forall l \in L,\ \forall s_{a,j,l} \in S_{j,l}$$

  under the constraints (1) and (2), and the other possible constraints of the original problem.

5.3 Illustrative example: application of IMO-DRSA

Let us apply the IMO-DRSA to the decision problem introduced in Section 4. Taking into account the evaluations of the projects with respect to considered criteria shown in Table 4 and the thresholds in Table 10, we get the evaluations in ordinal qualitative terms shown in Table 11, where WS, S, VS and ES are representing our satisfaction classes “weakly satisfactory”, “satisfactory”, “very satisfactory”, and “extremely satisfactory”, respectively.

In a perspective of location oriented multiobjective optimization, each portfolio is evaluated in terms of facilities at least satisfactory, at least very satisfactory and extremely satisfactory in the North and in the South. In the first iteration, the six representative strategies presented in Table 12 are shown to the DM, where $\mathcal{F}_{a,l}(\mathbf{x}) = \sum_{j \in J} F_{a,j,l}(\mathbf{x})$. For example $\mathcal{F}_{1,1}(\mathbf{x}) = \sum_{j \in J} F_{1,j,1}(\mathbf{x})$ is the number of all the facilities that have a contribution for each criterion at least satisfactory. In Table

Table 11: Qualitative ordinal evaluations on three criteria in each location of the eight facilities considered in the illustrative example.

Facilities	*EconomicImpact*		*SocialImpact*		*EnvironmentalImpact*	
	North	South	North	South	North	South
School	S	S	ES	ES	S	S
Leisure Centre	VS	VS	ES	ES	VS	S
Council Offices	WS	WS	S	S	S	S
Recycling Centre	ES	ES	ES	ES	ES	ES
Start Up Incubator	ES	ES	WS	WS	WS	WS
Healthcare Centre	WS	WS	WS	WS	VS	ES
Community Centre	S	S	ES	VS	S	VS
Social Housing	WS	S	ES	ES	WS	WS

Table 12: The set of non-dominated strategies presented to the DM in the first iteration.

Strategy	$\mathcal{F}_{1,1}$	$\mathcal{F}_{1,2}$	$\mathcal{F}_{2,1}$	$\mathcal{F}_{2,2}$	$\mathcal{F}_{3,1}$	$\mathcal{F}_{3,2}$	Class
$ST1$	12	0	11	0	5	0	*
$ST2$	0	14	0	9	0	6	Good
$ST3$	12	0	11	0	5	0	*
$ST4$	0	13	0	10	0	6	Good
$ST5$	8	2	7	1	6	1	*
$ST6$	1	12	1	9	0	6	Good

13 we report the corresponding strategies. Let us underline that to facilitate the understanding of the solution for the DMs, each strategy could be presented to the DM with some graphs representing, by means of histograms, the values of $\mathcal{F}_{a,l}(\mathbf{x})$ as shown in Barbati et al. (2017). For the sake of the space we do not report here these representations.

The DM is asked if among the strategies shown to her there is one that she considers as completely satisfactory. Since this is not the case, she was asked to select a set of strategies that can be considered as relatively good. Consequently, she indicated strategies ST2, ST4 and ST6. Applying DRSA to this preference information, the following decision rules were induced (among parentheses we provide the strategies supporting the corresponding rule):

Rule 1.1: if $\mathcal{F}_{2,2}(\mathbf{x}) \geq 9$, then strategy $\mathbf{x}$ is "good", $(ST2, ST4, ST6)$

(if there are at least 9 projects very satisfactory or better in location South with respect to all criteria, then the portfolio is good);

Rule 1.2: if $\mathcal{F}_{3,2}(\mathbf{x}) \geq 6$, then strategy $\mathbf{x}$ is "good", $(ST2, ST4, ST6)$

(if there are at least 6 projects extremely satisfactory in location South with respect to all criteria, then the portfolio is good);

Table 13: Position and period of the activated facilities for each strategy.

Facilities	$ST1$		$ST2$		$ST3$		$ST4$		$ST5$		$ST6$	
	Location	Period	Location	Period	Location	Period	Location	Period	Location	Period	Location	Period
School	North	3	South	3	North	3	South	3	North	2	South	3
Leisure Centre	North	0	South	0	North	0	South	0	-	-	South	0
Council Offices	-	-	South	2	-	-	-	-	North	4	North	4
Recycling Centre	North	1	South	0	North	1	South	0	North	0	South	0
Start Up Incubator	North	4	-	-	North	0	-	0	North	3	-	0
Healthcare Centre	North	2	-	-	North	2	South	2	North	0	North	2
Community Centre	North	0	South	1	North	0	South	1	South	0	South	1
Social Housing	-	-	-	-	-	-	-	-	-	-	-	-

Table 14: The set of non-dominated strategies presented to the DM in the second iteration.

Strategy	$\mathcal{F}_{1,1}$	$\mathcal{F}_{1,2}$	$\mathcal{F}_{2,1}$	$\mathcal{F}_{2,2}$	$\mathcal{F}_{3,1}$	$\mathcal{F}_{3,2}$	Class
$ST1'$	2	12	1	7	0	3	*
$ST2'$	0	14	0	9	0	6	Good
$ST3'$	2	12	2	6	1	2	*
$ST4'$	0	13	0	10	0	6	Good
$ST5'$	1	12	1	8	1	5	*
$ST6'$	1	12	1	9	0	6	Good

Rule 1.3: if $\mathcal{F}_{1,2}(\mathbf{x}) \geq 12$, then strategy $\mathbf{x}$ is "good", $(ST2, ST4, ST6)$

(if there are at least 12 projects very satisfactory or better in location South with respect to all criteria, then the portfolio is good).

The DM selected Rule 1.3 as the most representative for her current aspirations, and the following constraint was added to the original optimization problem:

$$\sum_{j \in J} F_{1,j,2}(\mathbf{x}) \geq 12.$$

Then, the second sample of weakly non-dominated strategies (shown in Table 14) was generated and presented to the DM. For the sake of the space we do not report the correspondent strategies.

Again, the DM is asked if among the strategies shown to her there is one that she considers as completely satisfactory. Since this is not the case, she was asked to select a set of strategies that can be considered as relatively good. She indicated the strategies apart from ST2′, ST4′, ST6′. Applying DRSA to this preference information the following decision rules were induced (among parentheses we provide the strategies supporting the corresponding rule):

Rule 2.1: if $\mathcal{F}_{2,2}(\mathbf{x}) \geq 9$, then strategy $\mathbf{x}$ is "good", (ST2′, ST′4, ST′6)

Table 15: A set of non-dominated strategies presented to the DM in the third iteration.

Strategy	$\mathcal{F}_{1,1}$	$\mathcal{F}_{1,2}$	$\mathcal{F}_{2,1}$	$\mathcal{F}_{2,2}$	$\mathcal{F}_{3,1}$	$\mathcal{F}_{3,2}$	Class
$ST1''$	1	12	1	9	0	6	Good
$ST2''$	0	14	0	9	0	6	*
$ST3''$	1	12	1	9	0	6	*
$ST4''$	0	13	0	10	0	6	*
$ST5''$	0	14	0	9	0	6	*
$ST6''$	0	13	0	10	0	6	*

(if there are at least 9 projects very satisfactory or better in location South with respect to all criteria, then the portfolio is good);

Rule 2.2: if $\mathcal{F}_{3,2}(\mathbf{x}) \geq 6$, then strategy $\mathbf{x}$ is "good", (ST2′, ST′4, ST6′)

(if there are at least 6 projects extremely satisfactory in location South with respect to all criteria, then the portfolio is good);

Rule 2.3: if $\mathcal{F}_{1,2}(\mathbf{x}) \geq 13$, then strategy $\mathbf{x}$ is "good", (ST2′, ST4′)

(if there are at least 13 projects very satisfactory in location South with respect to all criteria, then the portfolio is good).

The DM selected Rule 2.1 as the most representative for her current aspirations, and thus, the following constraint was added to the original optimization problem and to the constraints added to the previous interaction:

$$\sum_{j \in J} F_{2,j,2}(\mathbf{x}) \geq 9.$$

Then, the third sample of weakly non-dominated strategies shown in Table 15 was generated and presented to the DM.

At this point the DM declares to be satisfied by the strategy ST1″ and the procedure stops.

6 Uncertainty and plurality of stakeholders

Two features that affect many real world problems are related to the uncertainty of the performances expected from activation of facilities Mild et al. (2015); Vilkkumaa et al. (2018) and to the presence of a plurality of stakeholders Salo and Hämäläinen (2001). When the criterion values are considered stochastic variables, several tools can be employed as, for example, the stochastic multicriteria acceptability analysis Lahdelma and Salminen (2009), or the interval stochastic variables as in Jiang et al. (2018).

In the following we introduce these further elements in our model.

6.1 Uncertainty

We model the uncertainty related to the performances of facilities from I with respect to criteria from J taking into account a set of states of nature related to the period $t \in T$ and to the states of nature realized in previous periods. Therefore we denote by

$$s_{(t,h_1,\ldots,h_t)}, t \in T - \{0\}$$

a state of nature taking place in period t in the sequence of previous states of nature,

$$s_{(1,h_1)}, s_{(2,h_1,h_2)}, \ldots, s_{(t-1,h_1,h_2,\ldots,h_{t-1})}.$$

For all $t \in T - \{0\}$ and for all path $s_{(1,h_1)}, s_{(2,h_1,h_2)}, \ldots, s_{(t-1,h_1,h_2,\ldots,h_{t-1})}$, let us denote by

$$p^C(s_{(t,h_1,h_2,\ldots,h_t)})$$

the probability of $s_{(t,h_1,h_2,\ldots,h_t)}$ conditioned to the path of previous states of nature

$$s_{(2,h_1,h_2)}, \ldots, s_{(t-1,h_1,h_2,\ldots,h_{t-1})}.$$

In other words, $p^C(s_{(t,h_1,h_2,\ldots,h_t)})$ is the probability of realization in period t of $s_{(t,h_1,h_2,\ldots,h_t)}$ if, in period $t-1$ state of nature $s_{(t,h_1,h_2,\ldots,h_{t-1})}$ is realized. Consequently, the (non conditioned) probability of the state of nature $s_{(t,h_1,\ldots,h_t)}$ is given by

$$p(s_{(t,h_1,\ldots,h_t)}) = p^C(s_{(1,h_1)}) \times p^C(s_{(2,h_1,h_2)}) \times \cdots \times p^C(s_{(t,h_1,\ldots,h_t)}).$$

For instance, let us consider the example in Figure 9 regarding the first facility, in the first location and for the first period. We have 2 periods and, for each period, we have two possible states of nature. Every state of nature is associated to a node of the diagram tree and the probability of each state of nature $p^C(s_{(t,h_1,h_2,\ldots,h_t)})$ is reported on the arc entering each node.

In this context we denote by $y_{ijlt}(s_{(t,h_1,\ldots,h_t)})$ the performance of facility $i \in I$ with respect to criterion $j \in J$ in location $l \in L$ at time $t \in T$ if the state of nature $s_{(t,h_1,\ldots,h_t)}$ is realized.

Taking into account the probabilities $p(s_{(t,h_1,\ldots,h_t)})$, we can compute the expected value of the performance of facility $i \in I$ with respect to criterion $j \in J$ in location $l \in L$ at time $t \in T - \{0\}$ as

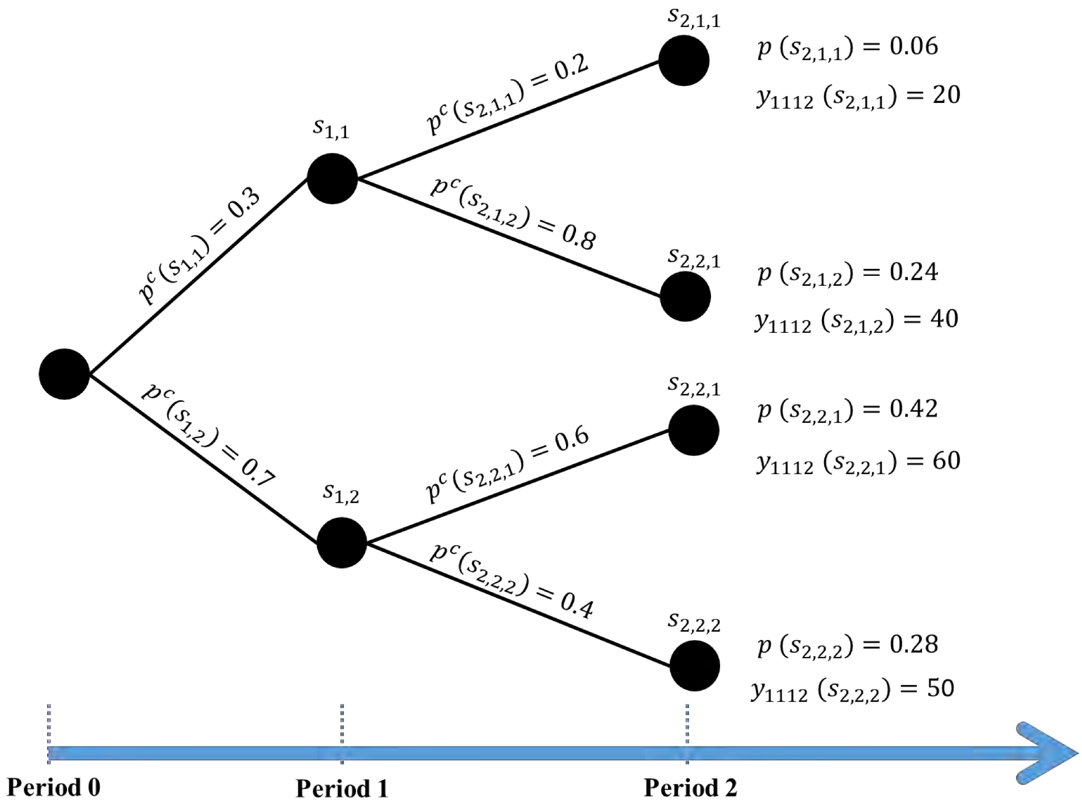

Figure 9: Example of a probability distribution of the performances.

follows:

$$E_p(y_{ijlt}) = \sum_{s_{(t,h_1,h_2,\ldots,h_t)} \in S_t} y_{ijlt}(s_{(t,h_1,\ldots,h_t)}) \times p(s_{(t,h_1,\ldots,h_t)}),$$

where S_t denotes the set of possible states of nature in period t.

In our didactic example, in Figure 9, the expected value $E_p(y_{ijl2})$ can be calculated as follows:

$$E_p(y_{ijl2}) = 0.06 \times 20 + 0.24 \times 40 + 0.42 \times 60 + 0.28 \times 50 = 50.$$

Given a strategy $\mathbf{x}$, the expected value $E_p(y_{ijlt}^{IJLT}(\mathbf{x}))$ of the performance of criterion $j \in J$ in period $t \in T - \{0\}$ from facility $i \in I$ is given by

$$E_p(y_{ijlt}^{IJLT}(\mathbf{x})) = \sum_{\tau=0}^{t-1} x_{il\tau} E_p(y_{ijlt}).$$

Analogously, the expected value of the discounted performance $y_{ijlt}^{IJLT}(\mathbf{x})$ is the following

$$E_p(\widehat{y}_{ijlt}^{IJLT}(\mathbf{x})) = E_p(y_{ijlt}^{IJLT}(\mathbf{x}))v(t) = \sum_{\tau=0}^{t-1} x_{il\tau} E_p(y_{ijlt})v(t).$$

Moreover, the expected value of all the other interesting values obtained from the values $y_{ijlt}^{IJLT}(\mathbf{x})$, can be easily obtained using $E_p(y_{ijlt}^{IJLT}(\mathbf{x}))$ instead of $y_{ijlt}^{IJLT}(\mathbf{x})$, as well as the corresponding discounted values can be obtained using $E_p(\widehat{y}_{ijlt}^{IJLT}(\mathbf{x}))$ instead of $\widehat{y}_{ijlt}^{IJLT}(\mathbf{x})$. For example the expected value of the the global performance of the strategy $\mathbf{x}$ with respect to criterion $j \in J$ in location $l \in L$ at time $t \in T - \{0\}$ is

$$E_p(y_{jlt}^{JLT}(\mathbf{x})) = \sum_{i \in I} E_p(y_{ijlt}^{IJLT}(\mathbf{x})) = \sum_{i \in I} \sum_{\tau=0}^{t-1} x_{il\tau} E_p(y_{ijlt})$$

and its discounted value is

$$E_p(\widehat{y}_{jlt}^{JLT}(\mathbf{x})) = \sum_{i \in I} E_p(\widehat{y}_{ijlt}^{IJLT}(\mathbf{x})) = \sum_{i \in I} \sum_{\tau=0}^{t-1} x_{il\tau} E_p(y_{ijlt})v(t).$$

In first approximation, the problem to be handled is to select the strategy $\mathbf{x}$ maximizing the expected value of the discounted overall performance taking into account all facilities $i \in I$, all criteria $j \in J$, all locations $l \in L$ and all periods $t \in T - \{0\}$ that is

$$E_p(\widehat{y}(\mathbf{x})) = \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} w_j x_{il\tau} E_p(y_{ijlt})v(t).$$

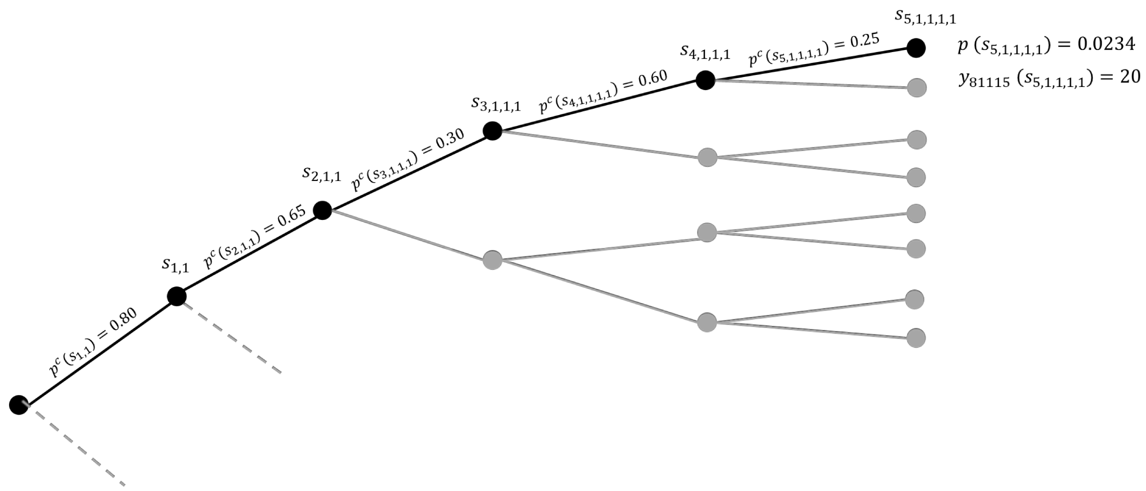

Figure 10: Probability distribution of the performances for the facility Social Housing with respect to the economic aspects.

Of course, also in this case one can handle the selection of the most preferred strategy by defining some compromise programming problem analogous to those ones illustrated in Section 5. One can use also some interactive multiobjective optimization method, such as the IMO-DRSA, again introduced in Section 4. In this perspective, to deal with uncertain performances and time preferences using DRSA, one can follow the approach proposed in Greco et al. (2010).

Illustrative Example: Uncertainty

In order to show how uncertainty can be taken into account with the proposed approach, we reconsider, for example, the performances related to the facility *Social Housing*, with respect to the economic criterion, having a probability distribution in both the locations with two possible alternative states of nature in each period. For the sake of space limit, we reported in Figure 10 only one branch of the tree. Following the path highlighted in bold black, we can compute $p(s_{(5,1,1,1,1,1)}) = 0.80 \times 0.65 \times 0.30 \times 0.60 \times 0.25 = 0.023$ to which the performance $y_{811}(s_{(5,1,1,1,1,1)}) = 76$ is associated. The other evaluations and the other probabilities are listed in Table 16.

The evaluations y_{ijl} of our illustrative example in Table 4 remain the same, apart from y_{811} and y_{812} changed in $E_p(y_{811}) = 73$ and $E_p(y_{812}) = 76$, respectively. Maximizing the expected value of the discounted overall performance $E_p(\hat{y}(\mathbf{x}))$ we obtain the most preferred solution shown in Figure 11. We can note that the facility Social Housing has to be activated in the first period; in fact, its economic evaluation is much improved in comparison to the not probabilistic scenario and this determines its entrance in the optimal strategy; nevertheless some very low evaluations of the facilities are taken into account also in our probabilistic scenario.

Table 16: Performances $y_{811}(s_{(t,h_1,...,h_t)})$ and corresponding probabilities $y_{812}(s_{(t,h_1,...,h_t)})$ for each possible state of nature in the final period .

State of Nature	y_{811}		y_{812}	
	Probabilities	Performances	Probabilities	Performances
$s_{(5,1,1,1,1,1)}$	0.0234	76	0.0234	89
$s_{(5,1,1,1,1,2)}$	0.0702	64	0.0702	92
$s_{(5,1,1,1,2,1)}$	0.01872	96	0.01872	84
$s_{(5,1,1,1,2,2)}$	0.04368	78	0.04368	95
$s_{(5,1,1,2,1,1)}$	0.0364	81	0.0364	78
$s_{(5,1,1,2,1,2)}$	0.1456	86	0.1456	93
$s_{(5,1,1,2,2,1)}$	0.1092	66	0.1092	17
$s_{(5,1,1,2,2,2)}$	0.0728	69	0.0728	99
$s_{(5,1,2,1,1,1)}$	0.00819	78	0.00819	96
$s_{(5,1,2,1,1,2)}$	0.04641	64	0.04641	88
$s_{(5,1,2,1,2,1)}$	0.01764	81	0.01764	12
$s_{(5,1,2,1,2,2)}$	0.01176	67	0.01176	78
$s_{(5,1,2,2,1,1)}$	0.0196	90	0.0196	69
$s_{(5,1,2,2,1,2)}$	0.0784	81	0.0784	87
$s_{(5,1,2,2,2,1)}$	0.049	67	0.049	79
$s_{(5,1,2,2,2,2)}$	0.049	95	0.049	94
$s_{(5,2,1,1,1,1)}$	0.00612	39	0.00612	15
$s_{(5,2,1,1,1,2)}$	0.02448	68	0.02448	92
$s_{(5,2,1,2,2,1)}$	0.00162	76	0.00162	15
$s_{(5,2,1,2,2,2)}$	0.00378	26	0.00378	77
$s_{(5,2,1,1,1,1)}$	0.0084	80	0.0084	69
$s_{(5,2,1,1,1,2)}$	0.0336	70	0.0336	67
$s_{(5,2,1,2,2,1)}$	0.0252	94	0.0252	93
$s_{(5,2,1,2,2,2)}$	0.0168	43	0.0168	12
$s_{(5,2,2,1,1,1)}$	0.00432	62	0.00432	75
$s_{(5,2,2,1,1,2)}$	0.01008	44	0.01008	88
$s_{(5,2,2,2,2,1)}$	0.00384	65	0.00384	77
$s_{(5,2,2,2,2,2)}$	0.00576	26	0.00576	10
$s_{(5,2,2,1,1,1)}$	0.00448	66	0.00448	48
$s_{(5,2,2,1,1,2)}$	0.01792	51	0.01792	87
$s_{(5,2,2,2,2,1)}$	0.01176	42	0.01176	15
$s_{(5,2,2,2,2,2)}$	0.02184	41	0.02184	96

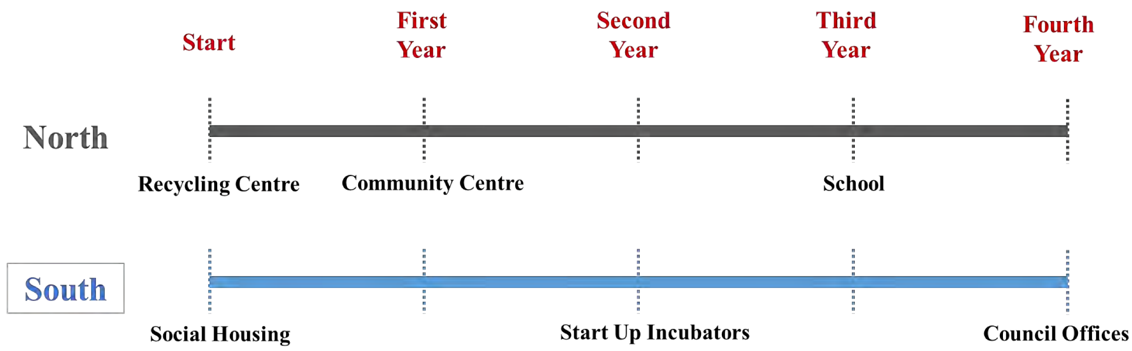

Figure 11: Optimal solution obtained by maximizing the expected value of the discounted overall performance $E_p(\hat{y}(\mathbf{x}))$.

6.2 Plurality of stakeholders

In planning problems we usually have a plurality of stakeholders such as municipality, building companies, association of citizens, trade union and so on Montibeller et al. (2009). Therefore it is reasonable to generalize our model to the presence of different perspectives and preferences expressed by different stakeholders. Here we present a basic approach of group decisions to our space-time model. Of course, more complex approaches can be considered. The basic idea is to assume a different weights vector for each stakeholder. Let us suppose that we have $K = \{1, \ldots, k, \ldots, b\}$ stakeholders. We consider weights $w_{jk} \geq 0$, such that $w_{1k} + \ldots + w_{qk} = 1$ and w_{jk} represents the weight assigned to criterion j by stakeholder k. We also introduce a central planner that defines a compromise solution giving a weight $z_k \geq 0$ representing the importance of each stakeholder, such that $z_1 + \ldots + z_b = 1$.

In this way, among the great plurality of performances $y^{sets}_{indices}(\mathbf{x})$ defined in Section 2 we can reformulate some of them and add others as follows:

- the overall performance of facility $i \in I$ in location $l \in L$ in period $t \in T - \{0\}$ taking into account all criteria, for stakeholder $k \in K$ is

$$y^{ILTK}_{iltk}(\mathbf{x}) = \sum_{j \in J} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl},$$

- the overall performance of the strategy $\mathbf{x}$ in location $l \in L$ in period $t \in T - \{0\}$ for stakeholder $k \in K$ is

$$y^{LTK}_{ltk}(\mathbf{x}) = \sum_{i \in I} \sum_{j \in J} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl}$$

- the overall performance of facility $i \in I$ in location $l \in L$ in period $t \in T - \{0\}$ taking into account all criteria and all stakeholders is

$$y_{ilt}^{ILT}(\mathbf{x}) = \sum_{k\in K} z_k \sum_{j\in J}\sum_{\tau=0}^{t-1} w_{jk}x_{il\tau}y_{ijl},$$

- the overall performance of facility $i \in I$ in period $t \in T - \{0\}$ considering all criteria $j \in J$ and all locations $l \in L$ for stakeholder $k \in K$ is

$$y_{itk}^{ITK}(\mathbf{x}) = \sum_{j\in J}\sum_{l\in L}\sum_{\tau=0}^{t-1} w_{jk}x_{il\tau}y_{ijl},$$

- the performance of facility $i \in I$ in location $l \in L$ for stakeholder $k \in K$ considering all criteria $j \in J$ and all periods $t \in T - \{0\}$ is

$$y_{ilk}^{ILK}(\mathbf{x}) = \sum_{j\in J}\sum_{t\in T-\{0\}}\sum_{\tau=0}^{t-1} w_{jk}x_{il\tau}y_{ijl},$$

- the overall performance of strategy $\mathbf{x}$ in period $t \in T - \{0\}$ for stakeholder $k \in K$ considering all facilities $i \in I$, all criteria $j \in J$ and all locations $l \in L$ is

$$y_{tk}^{TK}(\mathbf{x}) = \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{\tau=0}^{t-1} w_{jk}x_{il\tau}y_{ijl},$$

- the overall performances of strategy $\mathbf{x}$ in location $l \in L$ for stakeholder $k \in K$ considering all criteria $j \in J$ and all periods $t \in T - \{0\}$ is

$$y_{lk}^{LK}(\mathbf{x}) = \sum_{i\in I}\sum_{j\in J}\sum_{t\in T-\{0\}}\sum_{\tau=0}^{t-1} w_{jk}x_{il\tau}y_{ijl},$$

- the overall performance of the strategy $\mathbf{x}$ for all the stakeholders in location $l \in L$ at time $t \in T - \{0\}$ is

$$y_{lt}^{LT}(\mathbf{x}) = \sum_{k\in K} z_k \sum_{i\in I}\sum_{j\in J}\sum_{\tau=0}^{t-1} w_{jk}x_{il\tau}y_{ijl}$$

- the performance of facility $i \in I$ in period $t \in T - \{0\}$ considering all criteria $j \in J$ and all locations $l \in L$ for all the stakeholders is

$$y_{it}^{IT}(\mathbf{x}) = \sum_{k \in K} z_k \sum_{j \in J} \sum_{l \in L} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl},$$

- the performance of facility $i \in I$ in location $l \in L$ considering all criteria $j \in J$ and all periods $t \in T - \{0\}$ for all stakeholders $k \in K$ is

$$y_{il}^{IL}(\mathbf{x}) = \sum_{k \in K} z_k \sum_{j \in J} \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl},$$

- the performance of facility $i \in I$ and for stakeholder $k \in K$ with respect to all criteria $j \in J$, all location $l \in L$ and all periods $t \in T - \{0\}$ is

$$y_{ik}^{IK}(\mathbf{x}) = \sum_{j \in J} \sum_{l \in L} \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl},$$

- the overall performances of strategy $\mathbf{x}$ in location $l \in L$ considering all criteria $j \in J$, all periods $t \in T - \{0\}$ and all stakeholders $k \in K$ is

$$y_{l}^{L}(\mathbf{x}) = \sum_{k \in K} z_k \sum_{i \in I} \sum_{j \in J} \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl},$$

- the overall performance of strategy $\mathbf{x}$ for stakeholder $k \in K$ taking into account all facilities $i \in I$, all criteria $j \in J$, all locations $l \in L$ and all periods $t \in T - \{0\}$ is

$$y_{k}^{K}(\mathbf{x}) = \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl},$$

- the performance of facility $i \in I$ with respect to all criteria $j \in J$, all location $l \in L$, all periods $t \in T - \{0\}$ and all stakeholders $k \in K$ is

$$y_{i}^{I}(\mathbf{x}) = \sum_{k \in K} z_k \sum_{j \in J} \sum_{l \in L} \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl},$$

- the overall performance in period $t \in T - \{0\}$ considering all facilities $i \in I$, all criteria $j \in J$, all locations $l \in L$ and all stakeholders $k \in K$ is

$$y_t^T(\mathbf{x}) = \sum_{k \in K} z_k \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl},$$

- the overall performance taking into account all facilities $i \in I$, all criteria $j \in J$, all locations $l \in L$, all periods $t \in T - \{0\}$ and all stakeholders is

$$y(\mathbf{x}) = \sum_{k \in K} z_k \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T-\{0\}} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl}.$$

As before, in the first instance, the problem is to define the strategy $\mathbf{x}$ giving the maximum overall discounted performance $\widehat{y}(\mathbf{x})$ subject to the constraints of the problem such as the budget constraints and the activation constraints. However, the definition of several $\widehat{y}_{indices}^{sets}(\mathbf{x})$ can be an even richer dashboard that can be handled as a multiobjective optimization of performances $y_{indices}^{sets}(\mathbf{x})$ and $\widehat{y}_{indices}^{sets}(\mathbf{x})$ for multiple stakeholders. In addition, to search for the most preferred solution adopting the weighted approach, we can use a Compromise Programming approach dealing with multiple stakeholders (see for e.g., Phua and Minowa (2005)).

In our case we characterize our target as the optimal performance

$$\widehat{y}_k^{K^*} = \max_{\mathbf{x}} \widehat{y}_k^K(\mathbf{x}) = \sum_{i \in I} \sum_{j \in J} \sum_{t \in T-\{0\}} \sum_{\tau=0}^{t-1} w_{jk} x_{il\tau} y_{ijl} v(t)$$

that a strategy $\mathbf{x}$ can attain for stakeholder $k \in K$. Following Drezner et al. (2006), in order to get a balanced solution, we minimize the maximum deviation Δ_k^K, on the set of stakeholders $k \in K$, defined as

$$\Delta_k^K(\mathbf{x}) = \frac{\widehat{y}_k^{K^*} - \widehat{y}_k^K(\mathbf{x})}{\widehat{y}_k^{K^*}}.$$

Then, the distance of the strategy $\mathbf{x}$ from the ideal point is $\Delta^K(\mathbf{x}) = \max_{k \in K} \Delta_k^K(\mathbf{x})$. Consequently, $\Delta^k(\mathbf{x})$ is the objective to be minimizied to get the compromise solution searched for. This compromise optimisation strategy is particularly suitable in case the stakeholders need some reciprocal concessions between them in order to reach a consensus on a shared decision.

Illustrative Example: Plurality of Stakeholders

We apply the utility approach to the initial problem defined in Section 4 considering three stakeholders e.g., committees of the council with different interests: Development committee, Planning committee and Government committee. The weights w_{jk} and z_k are reported in Table 17. Using a

Table 17: Values for w_{jk} and z_k.

	Economic Impact	Social Impact	Environmental Impact	z_k
Planning committee	0.1	0.1	0.8	0.5
Development committee	0.1	0.2	0.7	0.4
Government committee	0.4	0.3	0.4	0.1

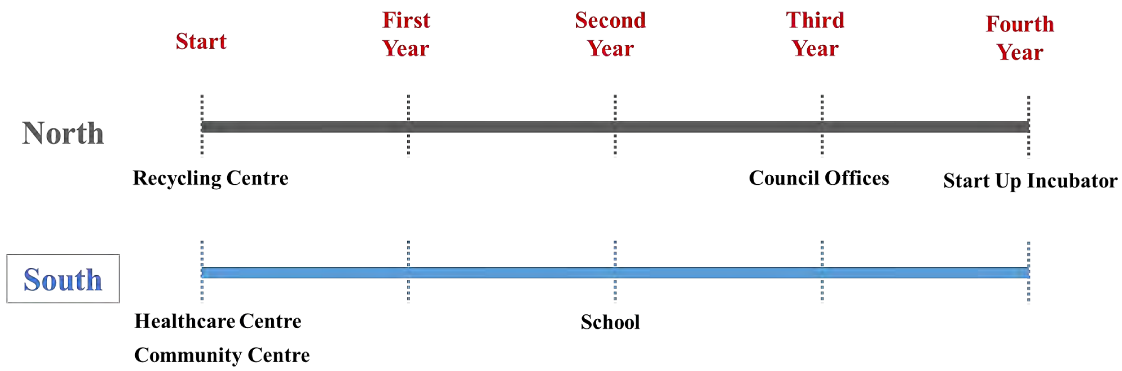

Figure 12: Optimal solution obtained by maximizing the overall performance aggregating the preferences of all the stakeholders with the weights of the central planner.

utility approach we find the optimal solution shown in Figure 12.

From Figure 13 we can see that Environmental Impact is largely more important than the other two criteria because it has a very high weight for the first two committees which are also the most important ones. This is even more evident if this optimal strategy is compared with the strategy obtained with a single DM presented in Section 4 and shown in Figure 3.

7 Conclusions

In this paper we proposed a general model for combinatorial optimization problems that is based on variables x_{ilt} which take value 1 if facility i is activated in location l at time t, and 0 otherwise. We believe that the model we are proposing has two main merits:

- from a more theoretical point of view, our model is in the crossroad of the three following main combinatorial optimization problems:
  - *knapsack problems*, because our model helps to choose the facilities to be activated as well as the knapsack algorithms determine the items to be selected,
  - *location problems*, because our model suggests where the selected facilities have to be activated,

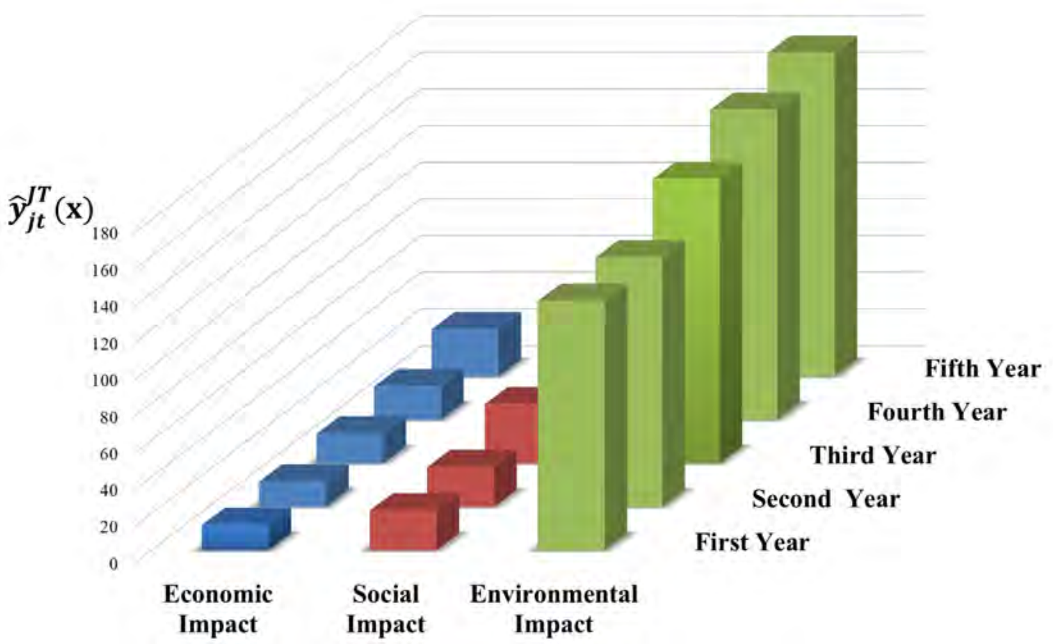

Figure 13: Time distribution of the performances for the optimal aggregating preferences of all stakeholders, with respect to the considered criteria.

  - *scheduling problems*, because our model suggests also when activating the selected facilities, possibly taking into account some precedence constraints;

- from a more application oriented point of view, our model permits to handle complex urban and territorial planning problems in a multiobjective perspective, taking into account a plurality of stakeholders and policy makers, considering also the uncertainty related to the outcomes of the decision to be taken.

Let us point out that our model not necessarily has to be applied to optimization problems with a combinatorial nature. Indeed, for example, the variable x_{ilt} can assume also the meaning of capital allocated to facility i in location l at time t. Therefore the most distinctive feature of our approach is the simultaneous consideration of space and time, so that we refer to our model in terms of space-time model. With respect to future developments of the research related to the model we are proposing, the two following points seem to us the most promising:

- efficient exact, approximate or heuristic algorithms and procedures to handle problems of big dimension with many facilities, many constraints and many locations,

- applications to real world decision problems in order to test the contribution that our model can give in terms of decision support and to define its possible areas of improvement and enhancement.

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